CONNECTICUT STATE DEPARTMENT OF EDUCATION

Math Practice Standards: Classroom Evidence

Presented by: Jennifer Michalek

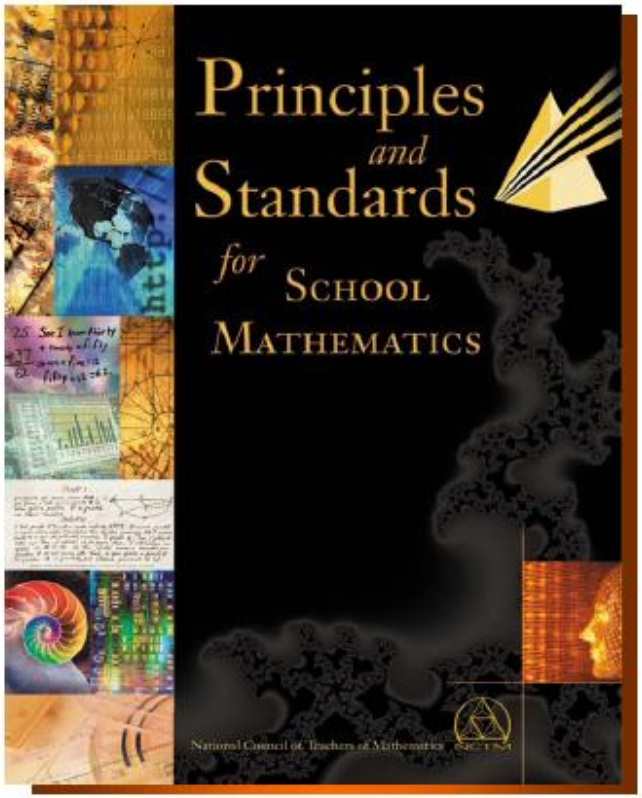
What's in the CCS-Math?

- The Standards for Mathematical Content
  - Define what students should understand and be able to do
  - K-8 Grade Level Domains
  - High School Conceptual Categories
- The Standards for Mathematical Practice
  - Describe HABITS OF MIND of a mathematically expert student
  - Recurring throughout the grades

Background of the Math Practice Standards

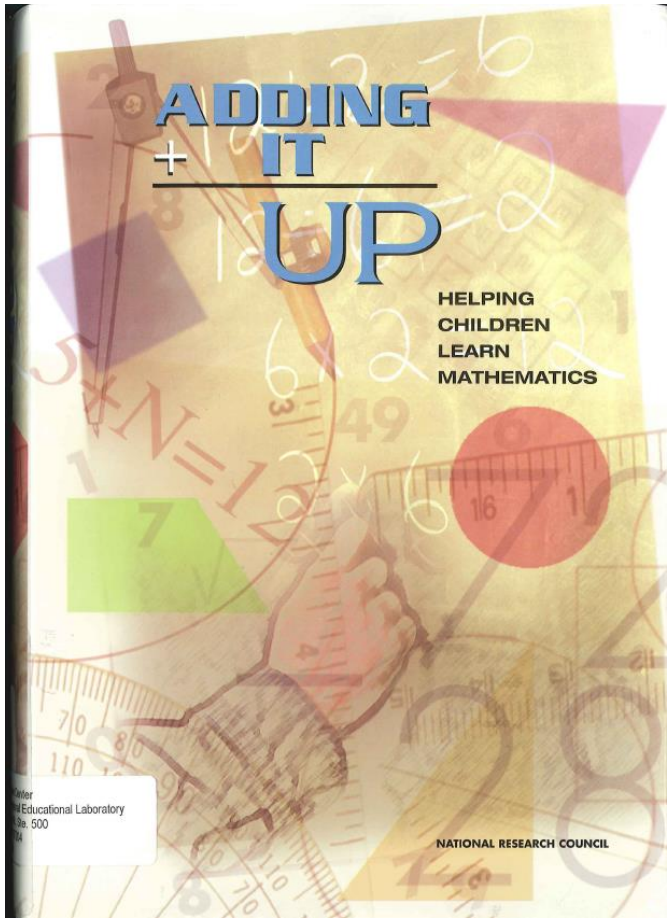
Draws from Two Sources

- *Principals and Standards for School Mathematics*



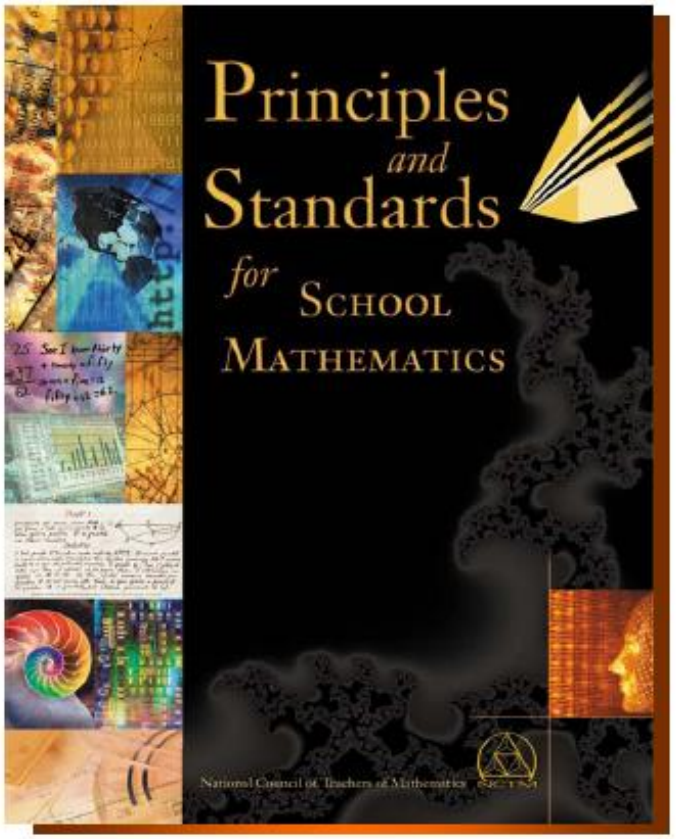
NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

- *Adding It Up*



NCTM Principles and Standards for School Mathematics

- Problem Solving
- Reasoning and Proof
- Communication
- Representation
- Connections



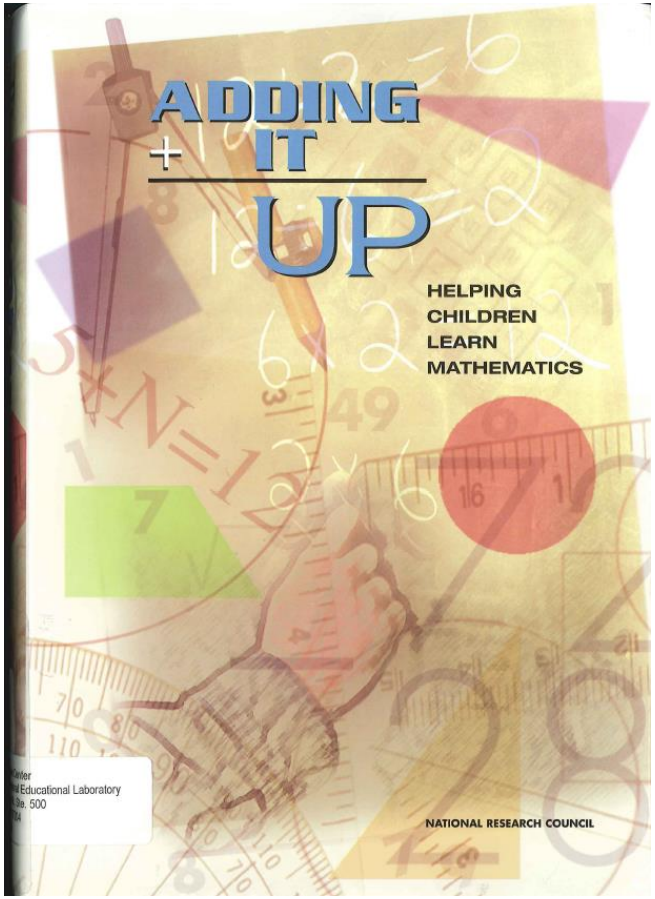
NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

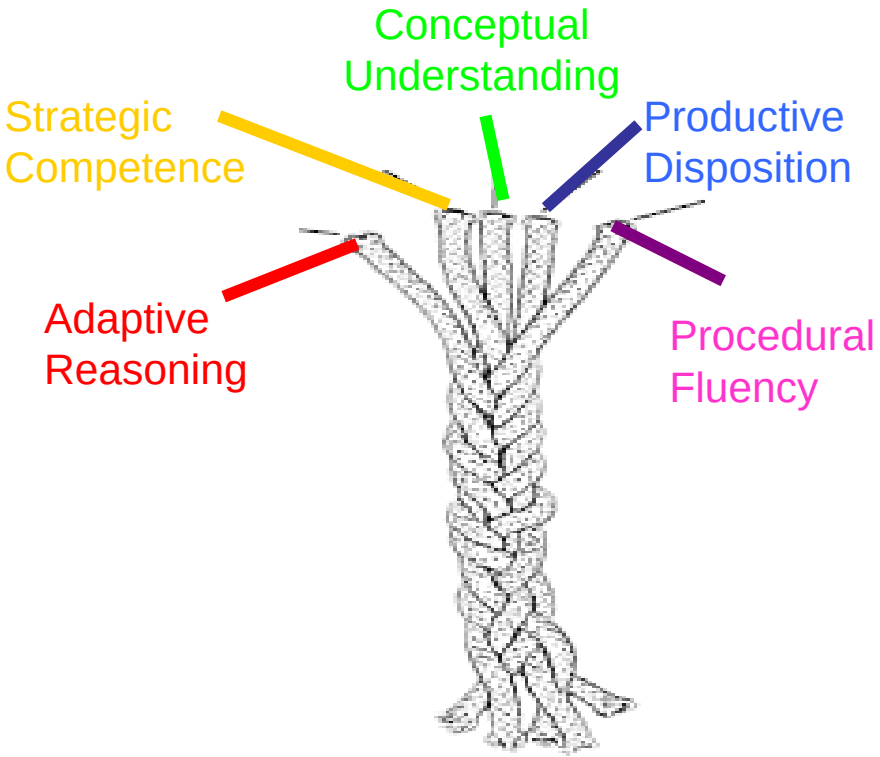
Process Standards

- ***Problem Solving -*** The process of applying a variety of appropriate strategies based on information provided, referenced, recalled, or developed.
- ***Reasoning and Proof –*** Making and investigating mathematical conjectures. Developing arguments and proofs.
- ***Communication –*** Organizing mathematical thinking coherently and clearly to peers, teachers and others. Using the language of math to express mathematical ideas precisely.
- ***Representation –*** Creating and using multiple representations to organize, record and communicate mathematical ideas. Using models and interpreting mathematical phenomena.
- ***Connections –*** Recognizing and using connections among math ideas as well as other subjects. Understanding how mathematical ideas interconnect and build on one another.

https://mathequality.wordpress.com/2012/06/26/nctm-process-standards-for-mathematics/

Adding It Up: Strands of Mathematical Proficiency





Strands of Mathematical Proficiency

- ***Conceptual Understanding*** – comprehension of mathematical concepts, operations, and relations
- ***Procedural Fluency*** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic Competence*** – ability to formulate, represent, and solve mathematical problems
- ***Adaptive Reasoning*** – capacity for logical thought, reflection, explanation, and justification
- ***Productive Disposition*** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Crosswalk

NCTM Process Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Representation
- Connections

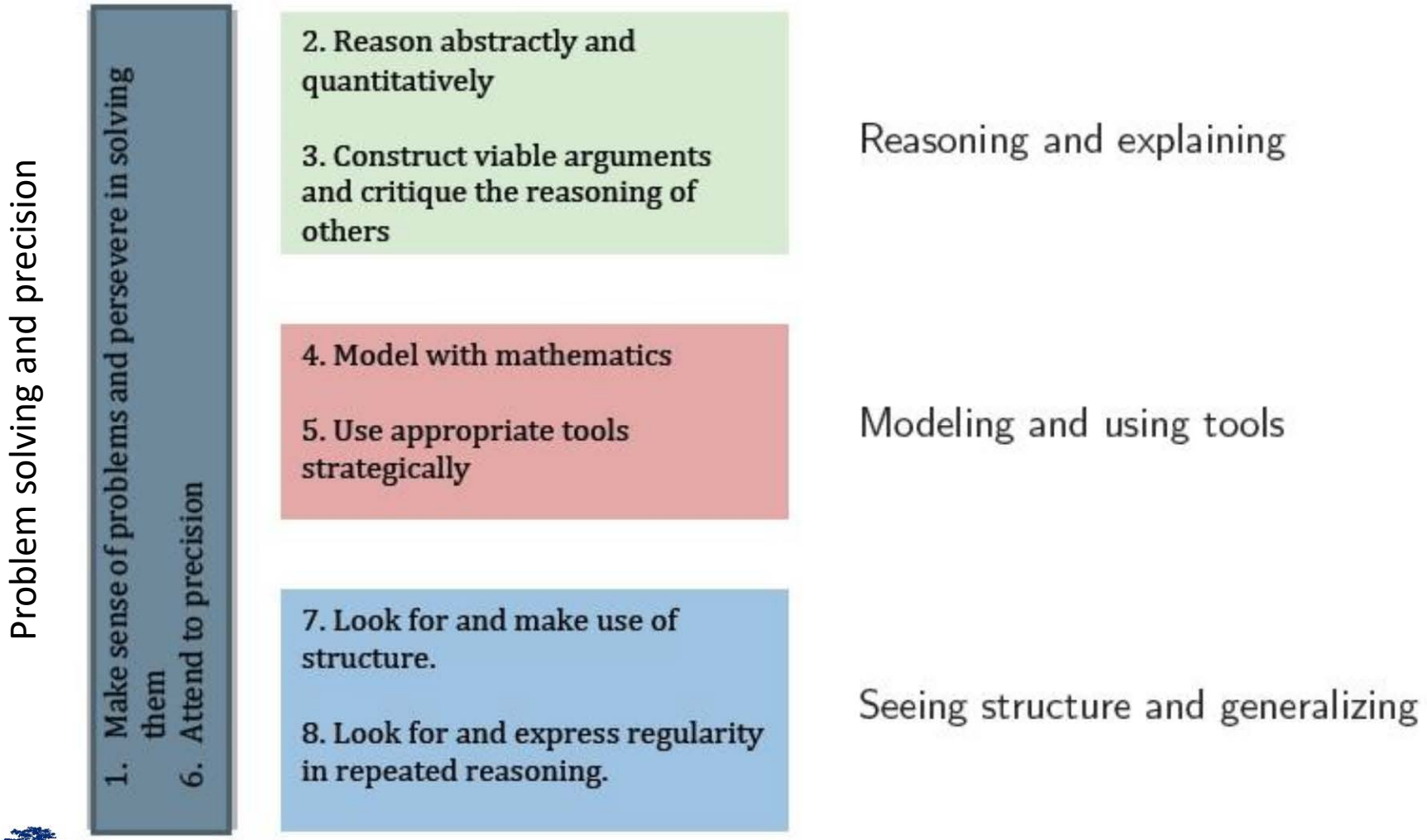
Strands of Mathematical Proficiency

- Conceptual Understanding
- Procedural Fluency
- Strategic Competence
- Adaptive Reasoning
- Productive Disposition

The Standards for Mathematical Practice

1. *Make sense of problems and persevere in solving them.*
2. *Reason abstractly and quantitatively.*
3. *Construct viable arguments and critique the reasoning of others.*
4. *Model with mathematics.*
5. *Use appropriate tools strategically.*
6. *Attend to precision.*
7. *Look for and make use of structure.*
8. *Look for and express regularity in repeated reasoning.*

Grouping the Standards



Bill McCallum, 2011

Understanding the Math Practice Standards

https://youtu.be/MP_dOFImZsI

Standards for Mathematical Practice Dual Nature

Standards for Mathematical Practice describe *mathematical content* students need to learn.

SP1. Make sense of problems

"… students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends."

Standards for Mathematical Practice describe the *nature of the learning experiences, thinking processes, habits of mind, and dispositions* that students need to develop a deep, flexible, and enduring understanding of mathematics.

SP1. Make sense of problems

"….they [students] analyze givens, constraints, relationships and goals. ….they monitor and evaluate their progress and change course if necessary. …. and they continually ask themselves "Does this make sense?"

The Beginning of the Practice Standards

Take a moment to examine the first three words of each of the 8 mathematical practices.

What do you notice?

Mathematically Proficient Students…

Practice Standards Actions

What are the *verbs* that illustrate the student actions for your assigned mathematical practice?

Circle, highlight or underline them for each practice.

Discuss with a partner:

What jumps out at you?

What few words summarize the actions needed for the practice?

What implications might each standard for mathematical practice have on classroom instruction?

The Standards for Mathematical Practice

SMP1: *Explain and make conjectures…*

SMP2: *Make sense of…*

SMP3: *Understand and use…*

SMP4: *Apply and interpret…*

SMP5: *Consider and detect…*

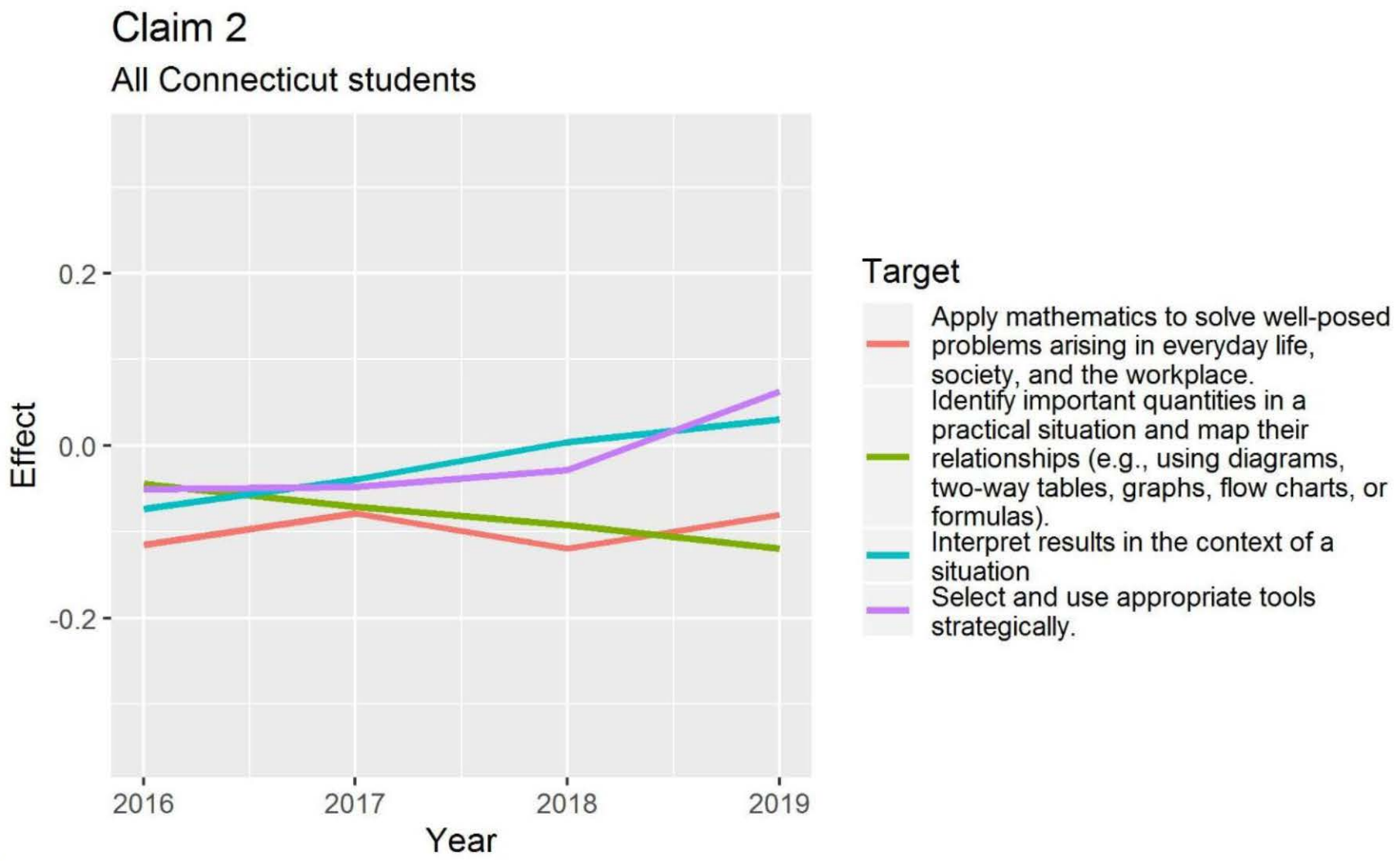
SMP6: *Communicate precisely to others…*

SMP7: *Discern and recognize…*

SMP8: *Notice and pay attention to…*

Assessing the Math Practice Standards

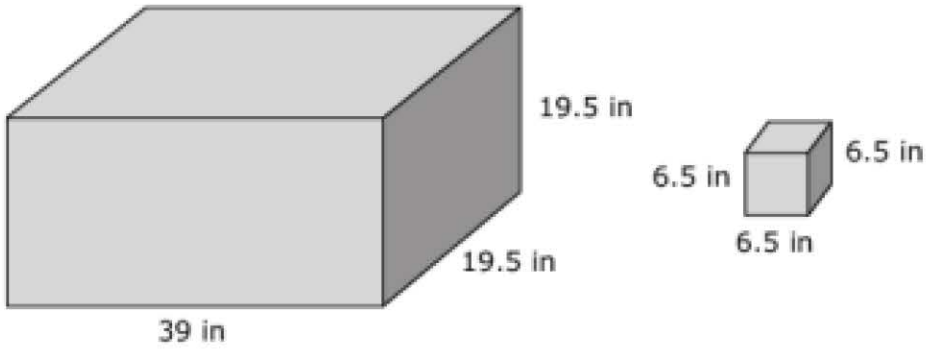
Claim 2 Data


Claim 2
All Connecticut students
0.2
0.0
-0.2
Effect
2016
2017
2018
2019
Year
Target
Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.
Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).
Interpret results in the context of a situation
Select and use appropriate tools strategically.

Claim 2 Assessment Item – Grade 6

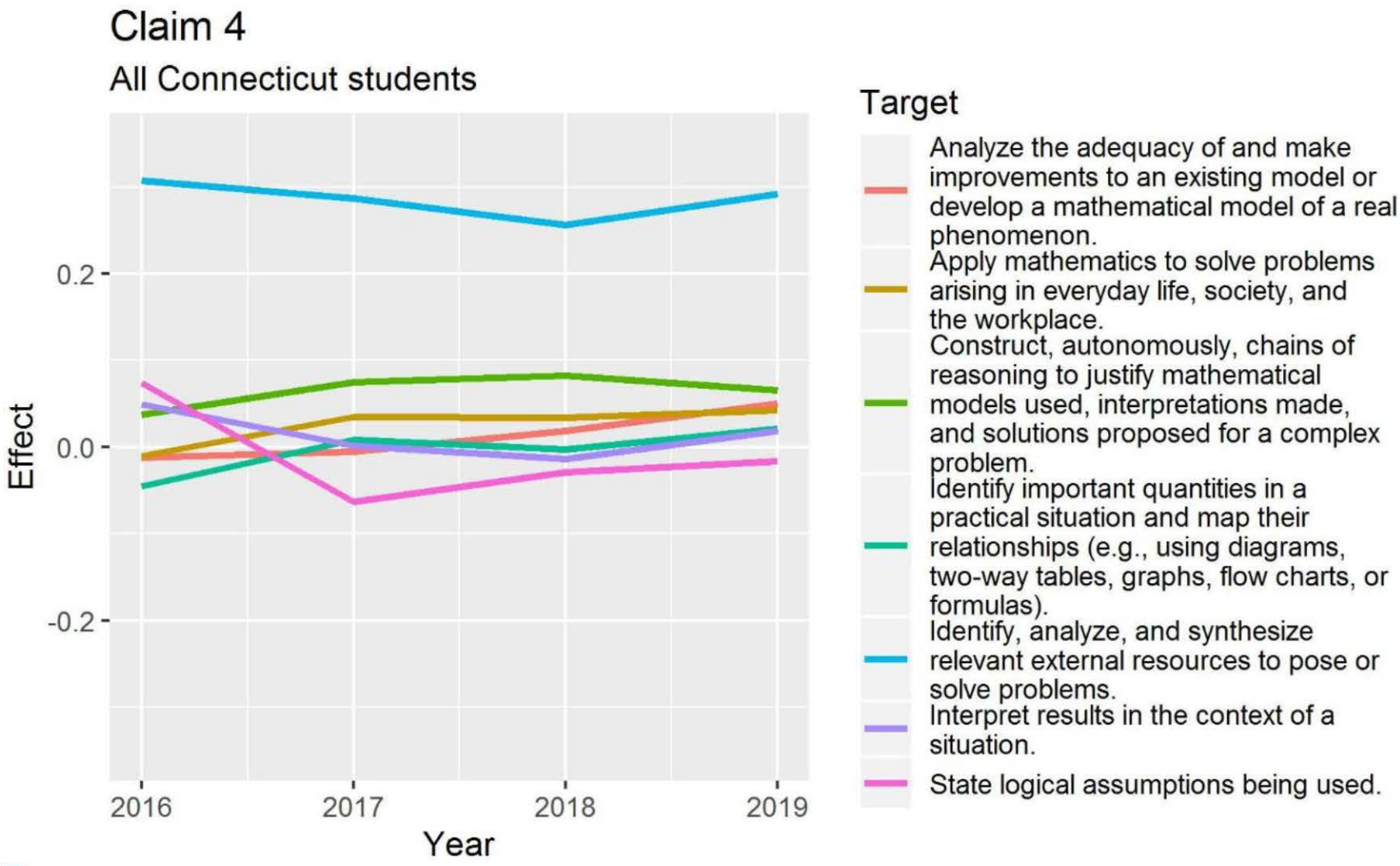
Cube-shaped boxes of tissue are shipped to stores in containers. The containers are rectangular prisms.

- The edges of each tissue box measure 6.5 inches.
- The dimensions of the shipping container are 19.5 inches by 39 inches by 19.5 inches.



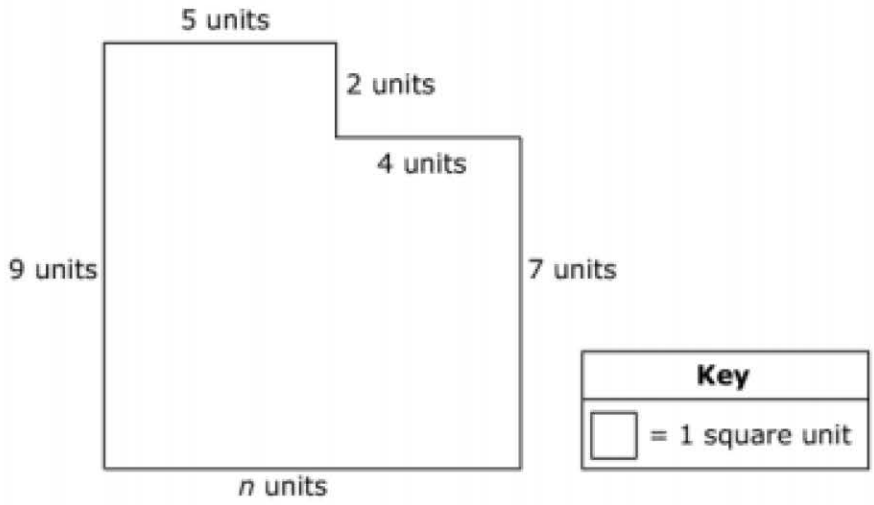
What is the greatest number of tissue boxes that will fit into one shipping container?

Claim 4 Data


Claim 4
All Connecticut students
Effect
0.2
0.0
-0.2
2016
2017
2018
2019
Year
Target
Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.
Apply mathematics to solve problems arising in everyday life, society, and the workplace.
Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.
Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).
Identify, analyze, and synthesize relevant external resources to pose or solve problems.
Interpret results in the context of a situation.
State logical assumptions being used.

Claim 4 Assessment Item – Grade 3

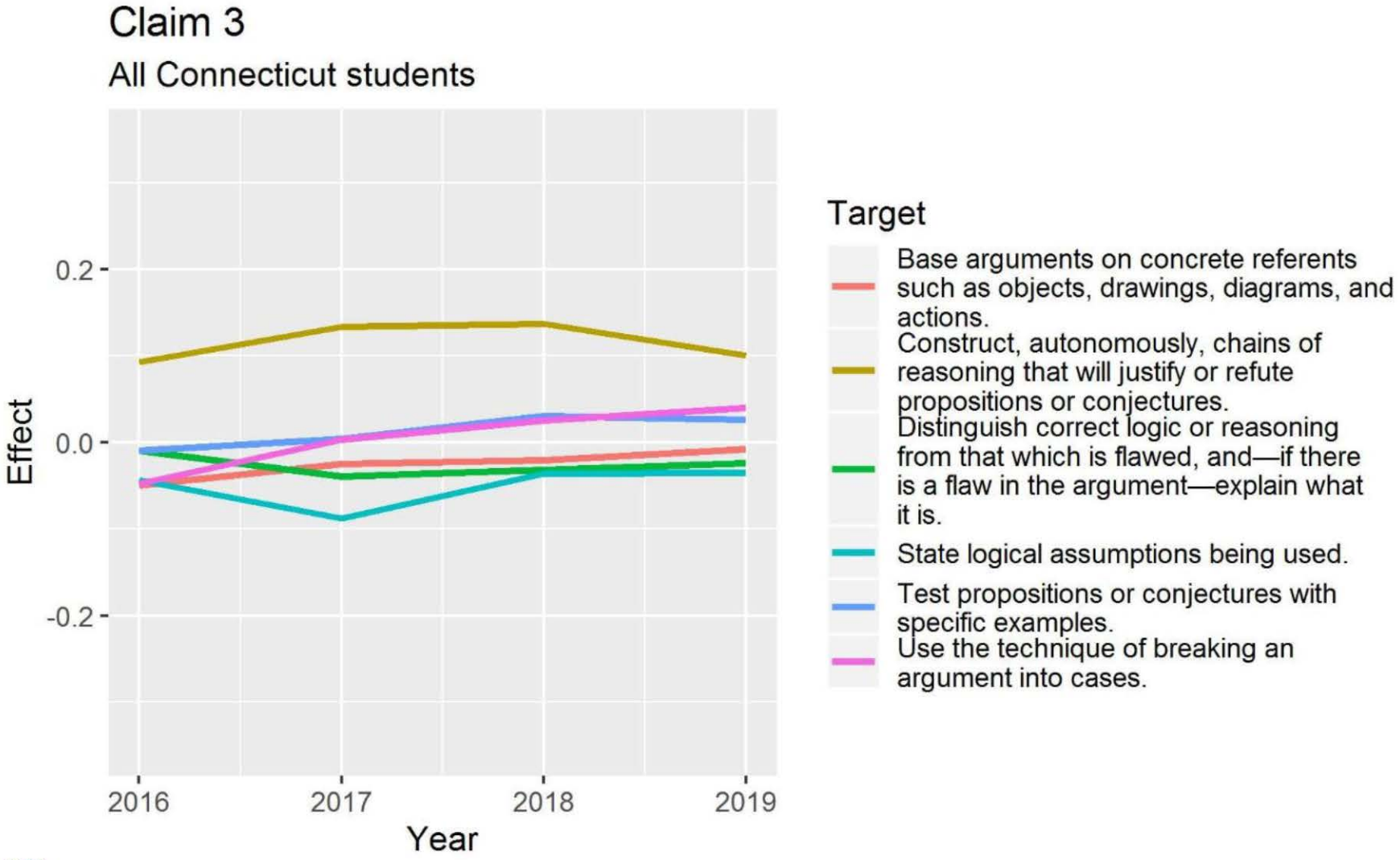
Juan draws a polygon with a perimeter of 36 units. He covers the area of the polygon with tiles that are each 1 square unit.



Part A: Enter an equation that could be used to find the value of *n* in the first response box.

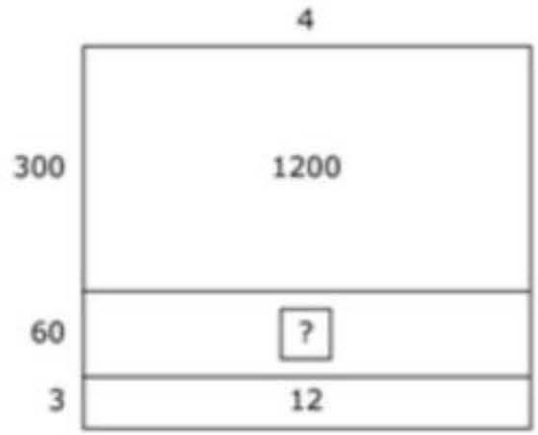
Part B: Enter the number of tiles Juan uses to cover the polygon in the second response box.

Claim 3 Data


Claim 3
All Connecticut students
Effect
0.2
0.0
-0.2
2016
2017
2018
2019
Year
Target
Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.
Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.
State logical assumptions being used.
Test propositions or conjectures with specific examples.
Use the technique of breaking an argument into cases.

Claim 3 Assessment Item – Grade 5

Jasmine solves the equation $\square \div 4 = 363$ using this area model.



Which statement explains how Jasmine should solve for the missing number in the model?

Ⓐ Jasmine should divide 60 by 4.

Ⓑ Jasmine should divide 1200 by 12.

Ⓒ Jasmine should multiply 3 times 60.

Ⓓ Jasmine should multiply 4 times 60.

Connecting Claims and Practices

- For Claims 2, 3, and 4 on the Smarter Balanced Assessment, the assessment targets are drawn from the CT Core Standards for Mathematical Practice.
- The assessment targets for Claims 2, 3, and 4 are the same across all tested grades.

Grade 4 Sample Assessment Item

Pablo solved a multiplication problem using two different methods. He made a mistake in either Method W or Method Z.

Method W

23 × 49

20 × 9 = 180
3 × 9 = 27
20 × 4 = 80
3 × 4 = + 12
299

Method Z

23 × 49

Area Model

	40	+ 9
20	800	180
+ 3	120	27

Rectangle Sections

1
800
120
180
+ 27
1,127

Identify the method where Pablo made a mistake and explain what he should do to correct it. Type your answer in the box below.

Deep Dive Into Implementation

Implementing the Practices

- **Content is the vehicle for engaging in the Standards for Mathematical Practice.**
  - ✓ Practices should be embedded in classroom instruction, discussions, activities and assessment and connected to content in meaningful ways.

- **The Standards for Mathematical Practice are not a checklist.**
  - ✓ Practices work together and a single student behavior could be thought of as exhibiting multiple practices at once.

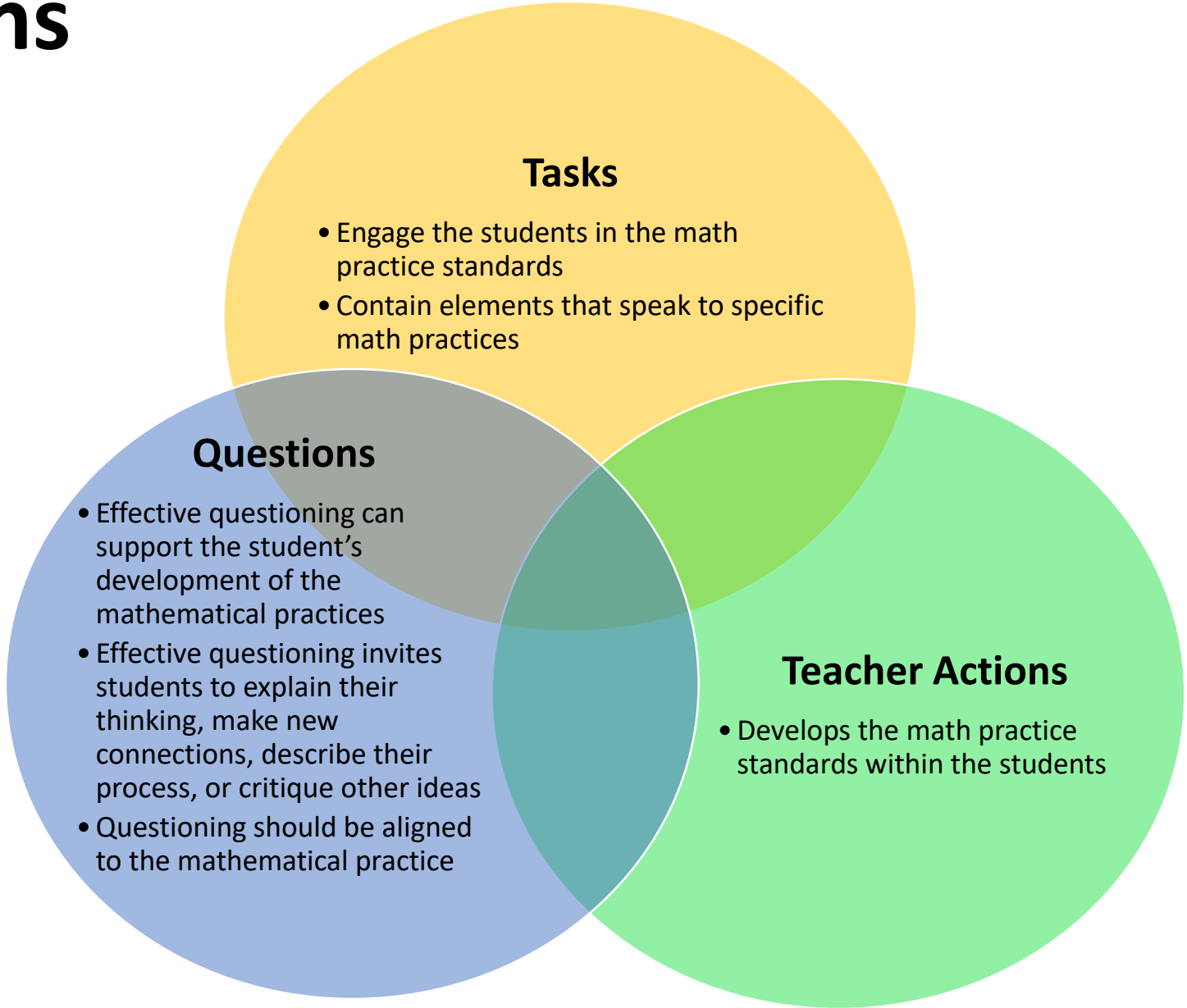
- **The Standards for Mathematical Practice look different across the grades as students grow in mathematical maturity.**
  - ✓ There is a progression of the math practice standards.

Successfully Implementing the Standards …..

Requires:

- Teacher knowledge of mathematics language and content.
- Teacher knowledge of how to promote student involvement in mathematical practices.
- Shifting students' focus from "answer getting" to solving problems.
- Establishing the classroom environment as a community of learners.

Lessons


Tasks
• Engage the students in the math practice standards
• Contain elements that speak to specific math practices
Questions
• Effective questioning can support the student's development of the mathematical practices
• Effective questioning invites students to explain their thinking, make new connections, describe their process, or critique other ideas
• Questioning should be aligned to the mathematical practice
Teacher Actions
• Develops the math practice standards within the students

What do tasks look like if the math practices are addressed?

Task Element MP1

MP 1: Make sense of problems and persevere in solving them

- Requires students to engage with conceptual ideas that underlie the procedures to complete the task and develop understanding.
- Requires cognitive effort - while procedures may be followed, the approach or pathway is not explicitly suggested by the task, or task instructions and multiple entry points are available.
- Encourages multiple representations, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations to develop meaning.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.

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Task Element MP2

MP 2: Reason Abstractly and quanitatively

- Includes questions that require students to attend to the meaning of quantities and their relationships, not just how to compute them.
- Consistently expects students to convert situations into symbols in order to solve the problem; and then requires students to explain the solution within a meaningful situation.
- Contains relevant, realistic content.

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Task Element MP3

MP 3: Construct viable arguments and critique the reasoning of others

- Is structured to bring out multiple representations, approaches, or error analysis.
- Embeds discussion and communication of reasoning and justification with others.
- Requires students to provide evidence to explain their thinking beyond merely using computational skills to find a solution.
- Expects students to give feedback and ask questions of others' solutions.

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Task Element MP4

MP 4: Model with mathematics

- Is structured so that students represent the problem situation and their solution symbolically, graphically, and/or pictorially (may include technological tools) appropriate to the context of the problem.
- Invites students to create a context (real-world situation) that explains numerical/symbolic representations.
- Asks students to take complex mathematics and make it simpler by creating a model that will represent the relationship between the quantities.

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Task Element MP5

MP 5: Use appropriate tools strategically

- Lends itself to multiple learning tools. (Tools may include; concrete models, measurement tools, graphs, diagrams, spreadsheets, statistical software, etc.)
- Requires students to determine and use appropriate tools to solve problems.
- Asks students to estimate in a variety of situations:
  - a task when there is no need to have an exact answer
  - a task when there is not enough information to get an exact answer
  - a task to check if the answer from a calculation is reasonable

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Task Element MP6

MP 6: Attend to precision

- Requires students to use precise vocabulary (in written and verbal responses) when communicating mathematical ideas.
- Expects students to use symbols appropriately.
- Embeds expectations of how precise the solution needs to be (some may more appropriately be estimates).

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Task Element MP7

MP 7: Look for and make use of structure.

- Requires students to look for the structure within mathematics in order to solve the problem. (i.e. – decomposing numbers by place value; working with properties; etc.)
- Asks students to take a complex idea and then identify and use the component parts to solve problems. i.e. Building on the structure of equal sharing, students connect the understanding to the traditional division algorithm. When "unit size" cannot be equally distributed, it is necessary to break down into a smaller "unit size"such as with long division
- Expects students to recognize and identify structures from previous experience(s) and apply this understanding in a new situation. i.e. 7 x 8 = (7 x 5) + (7 x 3) OR 7 x 8 = (7 x 4) + (7 x 4)…new situations could be, distributive property, area of composite figures, multiplication fact strategies. .

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Task Element MP8

MP 8: Look for and express regularity in repeated reasoning.

- Present several opportunities to reveal patterns or repetition in thinking, so students can make a generalization or rule.
- Requires students to see patterns or relationships in order to develop a mathematical rule.
- Expects students to discover the underlying structure of the problem and come to a generalization.
- Connects to a previous task to extend learning of a mathematical concept.

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What practices are evident?

Joel and Marisa are running for president at their middle school (grades 6-8). After the votes are in, Joel and Marisa are each convinced that they have won the election.

- Joel argues that he has won a larger percentage of the overall vote than Marisa so he should be the new president.
- Marisa argues that she has won a larger percentage than Joel of the 6th grade vote and the 7th grade vote. Since the majority of the grades voted for her, she should be the new president.

Is it possible that both Joel and Marisa are making accurate claims? Explain.

Finding Evidence in Tasks

Activity 5

What do teacher actions look like if they are attending to the math practices?

Teacher Actions MP1

MP 1: Make sense of problems and persevere in solving them

- Allows students time to initiate a plan; uses question prompts as needed to assist students in developing a pathway.
- Continually asks students if their plans and solutions make sense.
- Questions students to see connections to previous solution attempts and/or tasks to make sense of current problem.
- Consistently asks to defend and justify their solution by comparing solution paths.

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Teacher Actions MP2

MP 2: Reason Abstractly and quantitatively

- Asks students to explain the meaning of the symbols in the problem and in their solution.
- Expects students to give meaning to all quantities in the task.
- Questions students so that understanding of the relationships between the quantities and/or the symbols in the problem and the solution are fully understood.

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Teacher Actions MP3

MP 3: Construct viable arguments and critique the reasoning of others

- Encourages students to use proven mathematical understandings, (definitions, properties, conventions, theorems etc.), to support their reasoning.
- Questions students so they can tell the difference between assumptions and logical conjectures.
- Asks questions that require students to justify their solution and their solution pathway.
- Prompts students to respectfully evaluate peer arguments when solutions are shared.
- Asks students to compare and contrast various solution methods
- Creates various instructional opportunities for students to engage in mathematical discussions (whole group, small group, partners, etc.)

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Teacher Actions MP4

MP 4: Model with mathematics

- Demonstrates and provides students experiences with the use of various mathematical models.
- Questions students to justify their choice of model and the thinking behind the model.
- Asks students about the appropriateness of the model chosen.
- Assists students in seeing and making connections among models.

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Teacher Actions MP5

MP 5: Use appropriate tools strategically

- Demonstrates and provides students experiences with the use of various math tools. A variety of tools are within the environment and readily available.
- Questions students as to why they chose the tools they used to solve the problem.
- Consistently models how and when to estimate effectively, and requires students to use estimation strategies in a variety of situations.
- Asks student to explain their mathematical thinking with the chosen tool.
- Asks students to explore other options when some tools are not available.

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Teacher Actions MP6

MP 6: Attend to precision

- Consistently uses and models correct content terminology.
- Expects students to use precise mathematical vocabulary during mathematical conversations.
- Questions students to identify symbols, quantities and units in a clear manner.

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Teacher Actions MP7

MP 7: Look for and make use of structure.

- Encourages students to look for something they recognize and have students apply the information in identifying solution paths (i.e. composing/decomposing numbers and geometric figures, identifying properties, operations, etc.)
- Expects students to explain the overall structure of the problem and the big math idea used to solve the problem.

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Teacher Actions MP8

MP 8: Look for and express regularity in repeated reasoning.

- Asks what math relationships or patterns can be used to assist in making sense of the problem.
- Asks for predictions about solutions at midpoints throughout the solution process.
- Questions students to assist them in creating generalizations based on repetition in thinking and procedures.

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Finding Evidence in Teacher Actions

Activity 6

What questions solicit the use of the math practices?

Questions MP1

- How would you describe the problem in your own words?
- How would you describe what you are trying to find?
- What do you notice about...?
- What information is given in the problem?
- Describe the relationship between the quantities.
- Describe what you have already tried. What might you change?
- Talk me through the steps you've used to this point.
- What steps in the process are you most confident about?
- What are some other strategies you might try?
- What are some other problems that are similar to this one?
- How might you use one of your previous problems to help you begin?
- How else might you organize...represent...show...?

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Questions MP2

- What do the numbers used in the problem represent?
- What is the relationship of the quantities?
- How is _____ related to _____?
- What is the relationship between _____and _____?
- What does_____mean to you? (e.g. symbol, quantity, diagram)
- What properties might we use to find a solution?
- How did you decide in this task that you needed to use...? Could we have used
- another operation or property to solve this task? Why or why not?

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Questions MP3

- What mathematical evidence would support your solution?
- How can we be sure that...? / How could you prove that...? Will it still work if...?
- What were you considering when...?
- How did you decide to try that strategy?
- How did you test whether your approach worked?
- How did you decide what the problem was asking you to find? (What was unknown?)
- Did you try a method that did not work? Why didn't it work? Would it ever work?
  - Why or why not?
- What is the same and what is different about...?
- How could you demonstrate a counter-example?

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Questions MP4

- What number model could you construct to represent the problem?
- What are some ways to represent the quantities?
- What's an equation or expression that matches the diagram..., number line.., chart...,
- table..?
- Where did you see one of the quantities in the task in your equation or expression?
- What math do you know that you could use to represent this situation?
- What assumptions do you have to make to solve the problem?
- What formula might apply in this situation?

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Questions MP5

- What mathematical tools could we use to visualize and represent the situation?
- What information do you have?
- What do you know that is not stated in the problem?
- What approach are you considering trying first?
- What estimate did you make for the solution?
- In this situation would it be helpful to use...a graph..., number line..., ruler...,diagram..., calculator..., manipulative?
- Why was it helpful to use...?
- What can using a _____ show us that _____may not?
- In what situations might it be more informative or helpful to use...?

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Questions MP6

- What mathematical terms apply in this situation?
- How did you know your solution was reasonable?
- Explain how you might show that your solution answers the problem.
- Is there a more efficient strategy?
- How are you showing the meaning of the quantities?
- What symbols or mathematical notations are important in this problem?
- What mathematical language..., definitions..., properties can you use to explain...?
- How could you test your solution to see if it answers the problem?

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Questions MP7

- What observations do you make about...?
- What do you notice when...?
- What parts of the problem might you eliminate..., simplify...?
- What patterns do you find in...?
- How do you know if something is a pattern?
- What ideas that we have learned before were useful in solving this problem?
- What are some other problems that are similar to this one?
- How does this relate to...?
- In what ways does this problem connect to other mathematical concepts?

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Questions MP8

- Will the same strategy work in other situations?
- Is this always true, sometimes true or never true?
- How would we prove that...?
- What do you notice about...?
- What is happening in this situation?
- What would happen if...?
- Is there a mathematical rule for...?
- What predictions or generalizations can this pattern support?
- What mathematical consistencies do you notice?

https://achievethecore.org/aligned/wp-content/uploads/2016/06/Implementing-Standards-for-Mathematical-Practices-Updated-2016.pdf

“Look Fors”

Putting it Together

How do I know that all students have opportunities to exhibit mathematical practices while engaging with the content of the lesson?

- Are questions and problems posed that prompt students to share their developing thinking?
- What opportunities do students have to work with and practice grade level problems?
- What strategies are used to encourage collaboration among students?
- Do students explain their thinking?
- Do students talk about each other's thinking?
- Do students persist in solving challenging problems?
- Does the teacher connect student's informal language to precise mathematical language?

Teacher Indicators

Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

A. The teacher provides opportunities for all students to work with and practice course-level problems and exercises.

B. The teacher cultivates reasoning and problem solving by allowing students to productively struggle.

C. The teacher poses questions and problems that prompt students to explain their thinking about the content of the lesson.

D. The teacher creates the conditions for student conversations where students are encouraged to talk about each other's thinking.

E. The teacher connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas.

Student Indicators

Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

A. Students work with and practice course-level problems and exercises.

B. Students persevere in solving problems in the face of difficulty.

C. Students share their thinking about the content of the lesson beyond just stating answers.

D. Students talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.

E. Students use increasingly precise mathematical language and ideas.

Final Thoughts

An Illustration of the Practices

Activity 8


QUESTIONS

Thank You

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