Shifting Math Instruction: Rigor in the Classroom

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Learning Targets

- Participants will gain a deeper understanding of the CCS-Math instructional shifts of focus, coherence, and rigor.
- Participants will deepen their knowledge and understand the importance of Connecticut Core Standards (CCS) aligned instruction.
- Participants will gain a clear vision of how the CCS should look in practice within the mathematics classroom

The Shifts for CCS - Math

The Need for Change

- High college remediation rates
- Stagnant academic progress
- Larger gap with international peers

Three Instructional Shifts for CCS - Mathematics

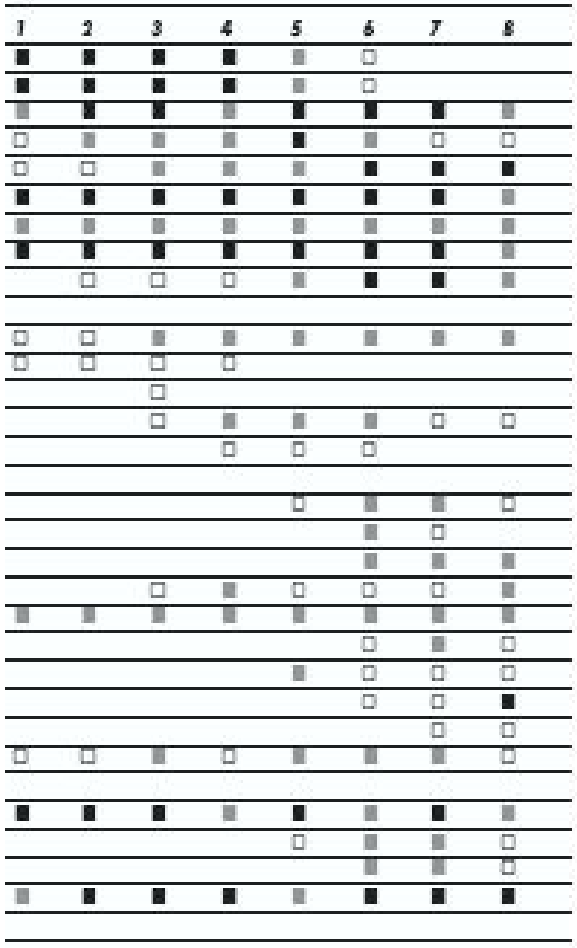
Focus on the Standards; teach less but for understanding.

Coherence – Carefully connect the learning within and across grades so that students can build new understanding on foundations built in previous years.

Rigor – Means a balance of solid conceptual understanding, procedural skill and fluency, and application of skills in problem solving situations.

The Shape of Math in A+ Countries

Mathematics topics intended at each grade by at least two-thirds of A+ countries



Mathematics topics intended at each grade by at least two-thirds of 21 U.S. states

[1] *Schmidt, Houang, & Cogan, "A Coherent Curriculum: The Case of Mathematics." (2002).*

Focus Through Domains

Table 1. The Pre-CCSSM Strand Picture of Mathematical Content

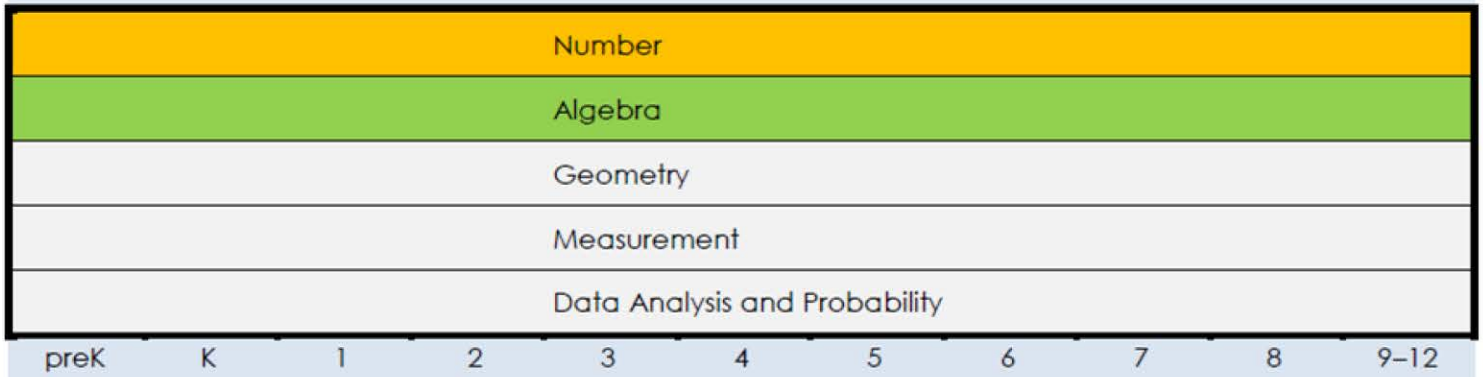
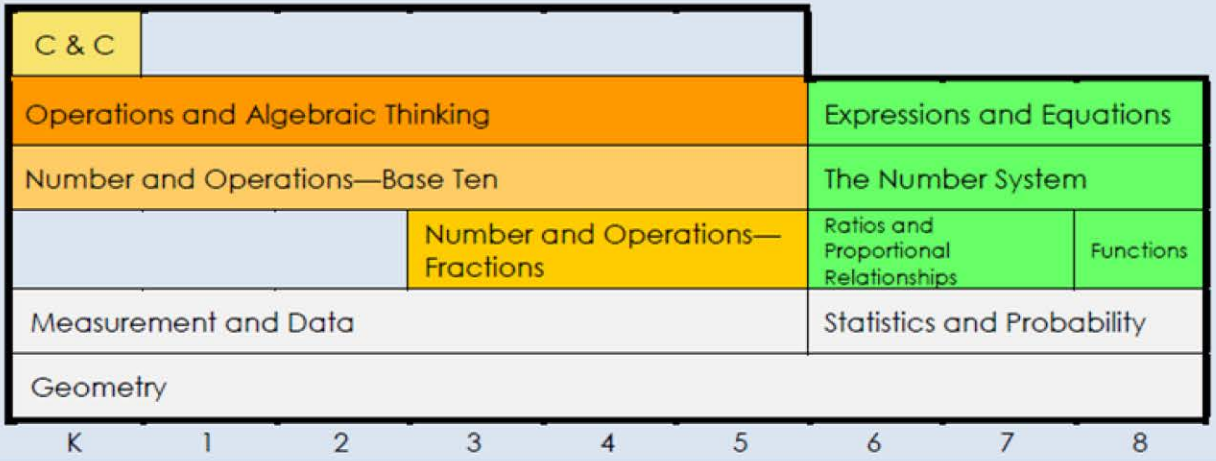


Table 2. The CCSSM Domain Picture of Mathematical Content



Shift 1: Focus

Focus strongly where the Standards focus

- ✓ Narrow the scope of content
- ✓ Focus deeply on what is emphasized in the Standards
- ✓ Move away from "mile wide, inch deep"
- ✓ Less topic coverage can be associated with higher scores on those topics covered

Levels of Focus

First level of focus: Knowing what is to be taught at each grade level and what is not.

- Serve as the foundation for the grade
- Essential mathematical ideas for each grade level
- Narrow the scope of content and deepen how time and energy is spent in the math classroom

Critical Areas at Each Grade

- 2 to 4 critical areas are identified at each grade from K-8
- Outline the essential mathematical ideas for each grade level
- Form a firm foundation on which to build concepts and procedures in later years

	How Did You Do?		
K	Compare numbers	Use tally marks	Understand meaning of addition and subtraction
1	Add and subtract within 20	Measure lengths indirectly and by iterating length units	Create and extend patterns and sequences
2	Work with equal groups of objects to gain foundations for multiplication	Understand place value	Identify line of symmetry in two dimensional figures
3	Multiply and divide within 100	Identify the measures of central tendency and distribution	Develop understanding of fractions as numbers
4	Examine transformations on the coordinate plane	Generalize place value understanding for multi-digit whole numbers	Extend understanding of fraction equivalence and ordering
5	Understand and calculate probability of single events	Understand the place value system	Apply and extend previous understandings of multiplication and division to multiply and divide fractions
6	Understand ratio concepts and use ratio reasoning to solve problems	Identify and utilize rules of divisibility	Apply and extend previous understandings of arithmetic to algebraic expressions
7	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers	Use properties of operations to generate equivalent expressions	Generate the prime factorization of numbers to solve problems
8	Standard form of a linear equation	Define, evaluate, and compare functions	Understand and apply the Pythagorean Theorem

Levels of Focus

Second level of focus: Knowing the major work of each grade.

- Not all content is emphasized equally
- Directly related to the critical areas
- Majority of the time should be dedicated to the major work of the grade

Cluster Emphases

Resource

Student Achievement Partners (SAP)

- Non-profit founded by three of the contributing authors of CCSSM
- Develops and makes available tools and resources free of charge
- Focus by grade level

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 3

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters ☐ Supporting Clusters ○ Additional Clusters

3.OA.A	■	Represent and solve problems involving multiplication and division.
3.OA.B	■	Understand properties of multiplication and the relationship between multiplication and division.
3.OA.C	■	Multiply and divide within 100.
3.OA.D	■	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
3.NBT.A	○	Use place value understanding and properties of operations to perform multi-digit arithmetic.
3.NF.A	■	Develop understanding of fractions as numbers.
3.MD.A	■	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
3.MD.B	☐	Represent and interpret data.
3.MD.C	■	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.D	○	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
3.G.A	☐	Reason with shapes and their attributes.

What is a “large majority of time”?

At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the Major Work of the grade.

- K-8 Publishers’ Criteria, Spring 2013, p. 8

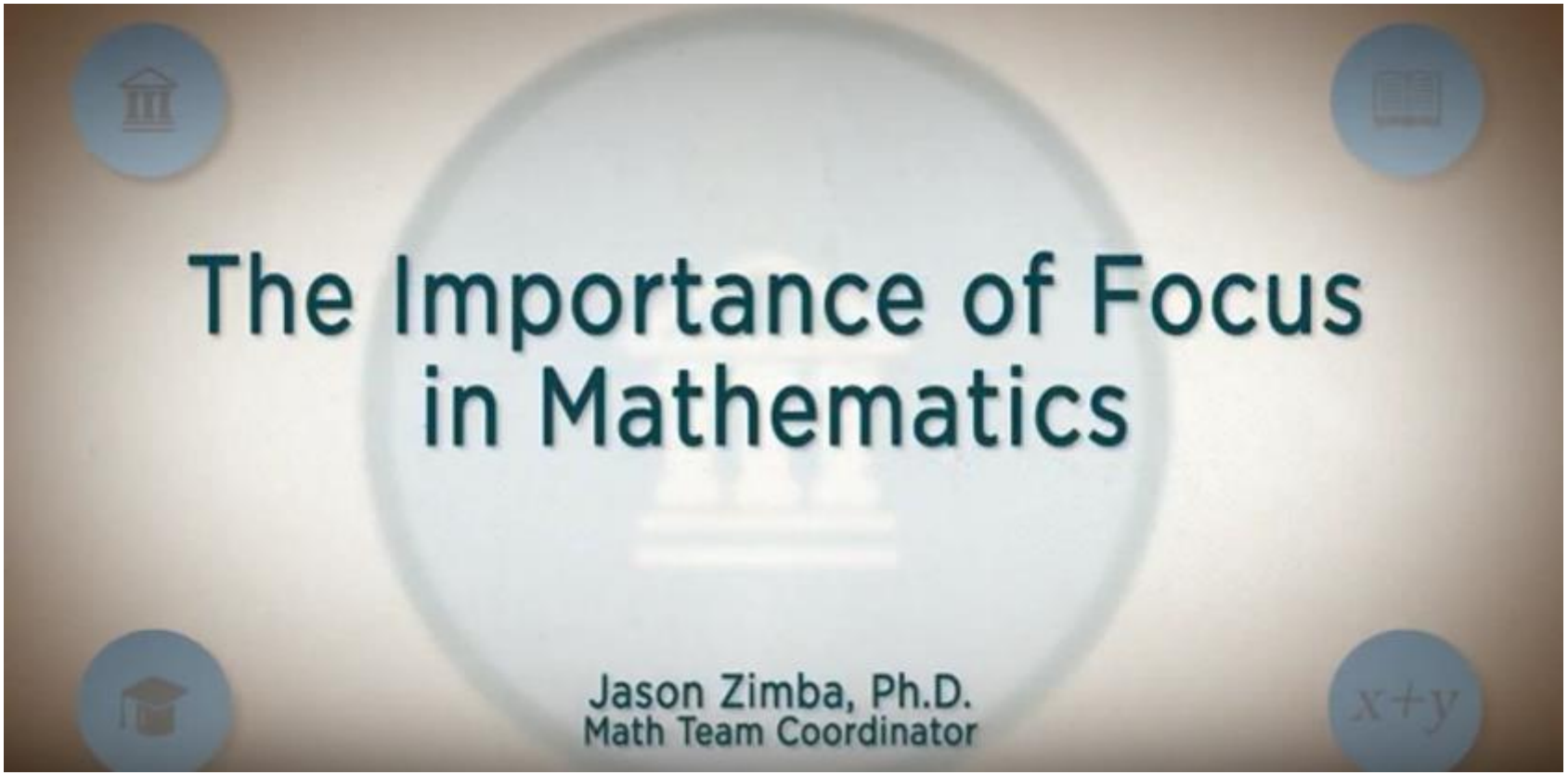
What does student learning look like if the teacher is focusing on the critical areas?

Look Fors

Grade	Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction - concepts, skills, and problem solving and place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra and linear functions

A Summary of Focus

Activity 2a



Three Instructional Shifts for CCS - Mathematics

Focus on the Standards; teach less but for understanding.

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Shift 2: Coherence

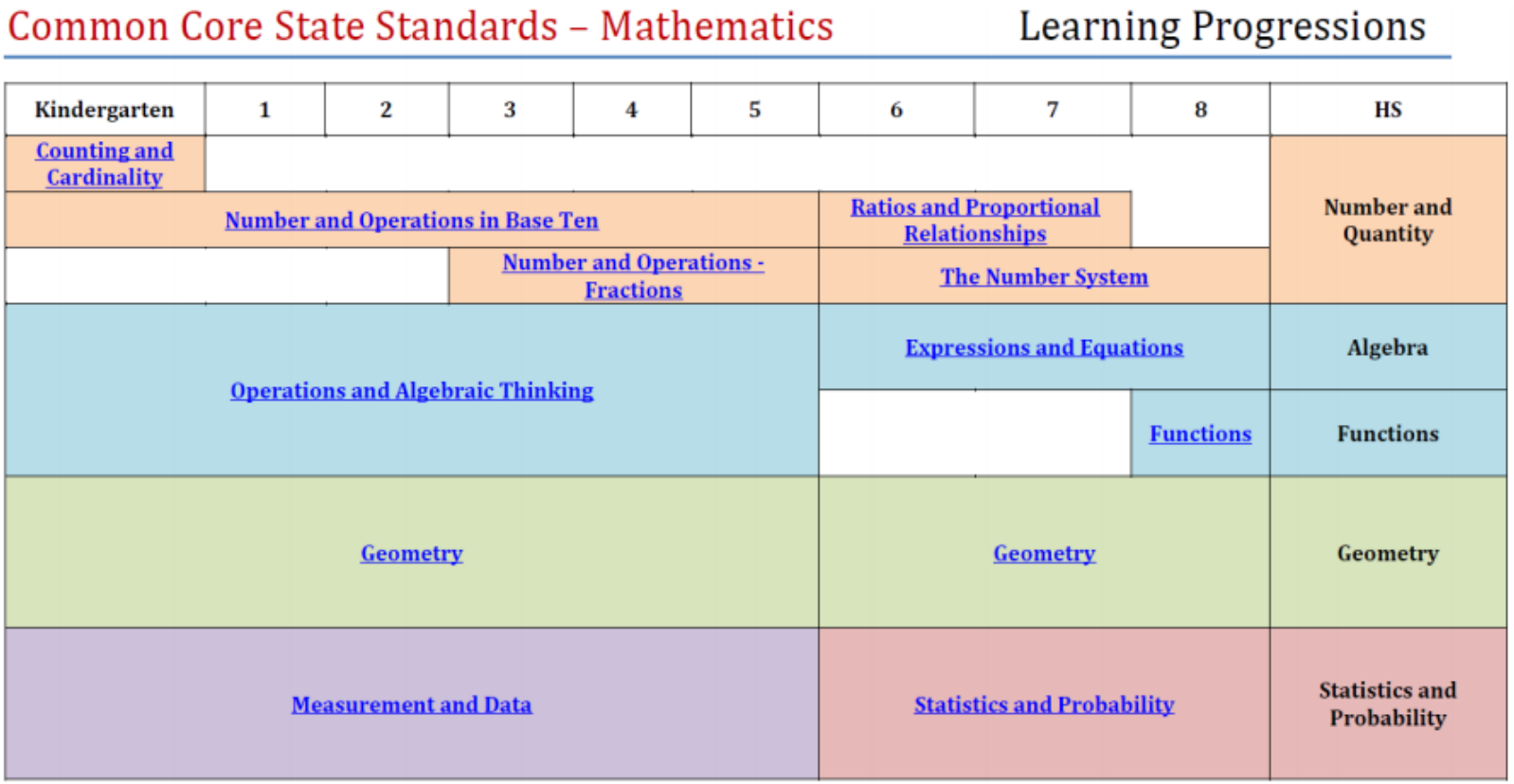
Think across grades and link to major topics within grades

- ✓ Connect learning within and across grades
- ✓ Each standard is not a new event, but an extension of previous learning
- ✓ Mathematics makes sense
- ✓ Based on the mathematical progressions

Why Coherence?

- The standards are designed around coherent progressions from grade to grade
- Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years.

Domains and Conceptual Categories K-12



Coherence is Built Into the Standards

Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

Two Levels of Coherence

- Coherence within a grade
  - Reinforce a major topic in a grade by utilizing a supporting topic
  - Meaningful introduction to topics in the same grade that complement each other
- Coherence across grades
  - Apply learning from previous grades to learn new topics
  - Progressions of mathematics that are meaningful and make sense

Coherence Within a Grade

Example: Data Representation

1.MD.C.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, **how many in each category, and how many more or less** are in one category than in another.

Coherence Within a Grade

Example: Statistics

8.SP.A.3 **Use the equation of a linear model** to solve problems in the context of bivariate measurement data, **interpreting the slope and intercept**. *For example, in a linear model for a biology experiment, interpret a slope of 1.5cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

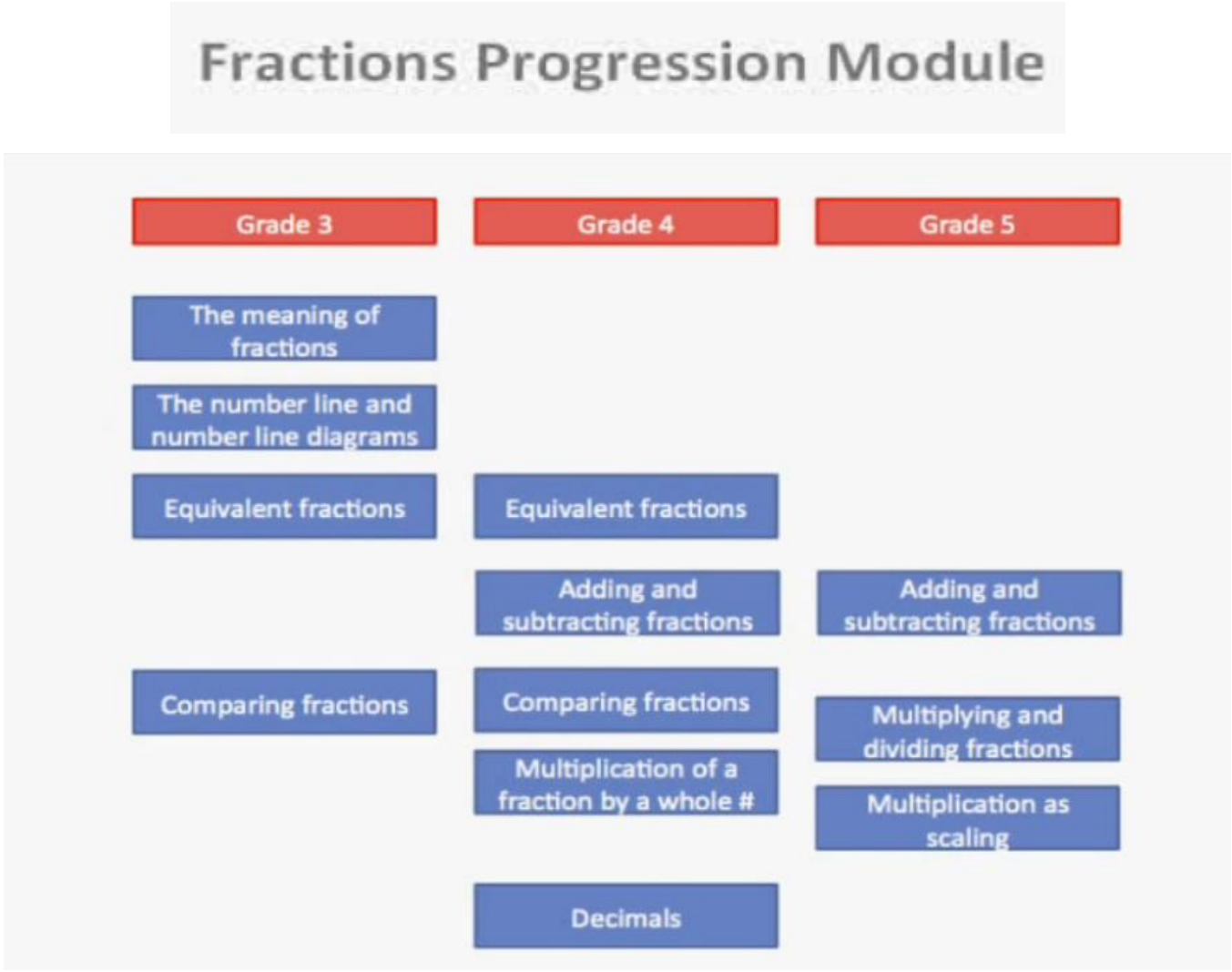
Coherence Across the Grades

K.OA.4: **For any number from 1 to 9, find the number that makes 10** when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	1.OA.6: **Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.** Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	2.OA.2: **Fluently add and subtract within 20 using mental strategies.** (Note: See standard 1.OA.6 for a list of mental strategies). By end of Grade 2, know from memory all sums of two one-digit numbers.

A Closer Look into the Domains….

3.NF.3: d. **Compare two fractions with the same numerator or the same denominator** by reasoning about their size.	4.NF.2: **Compare two fractions with different numerators and different denominators,** e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2.	5.NF.1: **Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions** in such a way as to produce an equivalent sum or difference of fractions with like denominators.

Putting It Into Practice



Coherence in the Middle Grades

6.EE.7: **Solve real-world and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* = *q*** for cases in which *p*, *q* and *x* are all nonnegative rational numbers.	7.EE.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. **Solve word problems leading to equations of the form *px* + *q* = *r* and *p*(*x* + *q*) = *r*, where *p*, *q*, and *r* are specific rational numbers.**	8.EE.8: Analyze and solve pairs of simultaneous linear equations. b. **Solve systems of two linear equations in two variables algebraically,** and estimate solutions by graphing the equations. Solve simple cases by inspection. c. Solve real-world and mathematical problems leading to two linear equations in two variables.

Coherence Map

- Build student understanding by linking together concepts within and across grades.
- Identify gaps in a student's knowledge by tracing a standard back through its logical pre-requisites.
- Visualize and understand how supporting standards relate to the major work of the grade.

What does student learning look like if the teacher is building coherence?

Look Fors

- Students experience a launch or initial review that anchors them before diving into new material.
- Students make comments like, “Oh! This is like when we…” or “It’s the same as when we used decimals” or “Last year we…”
- Students notice and ask about connections.
- Students can explain how some other concept they’ve studied is related to the current one.

Coherence Take-Aways

Coherence allows for:

- Linking work to major focus areas of the grade level.
- A natural progression of content from grade to grade, course to course.
- Support of mathematical connections between and among domains

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Rigor – Means a balance of solid conceptual understanding, procedural skill and fluency, and application of skills in problem solving situations.

Rigor: The three legged stool

Conceptual understanding

Procedural skill and fluency

Application

Shift 3: Rigor

In major topics, pursue conceptual understanding, procedural skill and fluency, and application

✓The CCSSM require a balance of:

-Conceptual understanding

-Procedural skill and fluency

-Application in problem-solving

✓Equal intensity in time, activities, and resources

Conceptual Understanding

Conceptual Understanding

- Teach more than “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives
- Conceptual understanding is about big ideas in math
- Students are able to see math as more than a set of mnemonics or discrete procedures
- Conceptual understanding is produced through rich problems, hands-on activities, skillful questioning, student discussion, etc.
- Conceptual understanding supports the other aspects of rigor (procedural skill and fluency, and application)

Building Conceptual Understanding

8.EE.B Understand the connections between proportional relationships, lines, and linear equations.

- The language in the standard speaks to conceptual understanding.
- Students need to understand more than just a process of creating a table or graphing a line.
- Students need to compare proportional relationships presented in different ways.

Questioning

- High-quality problems provide opportunities for discussion
- Good quality questions are easy to discuss
- Questions should elicit a range of responses from students which will enable class discussions to lead to conceptual understanding of a topic
- Questions do not have to be complex, but need to elicit student thinking about the conceptual understanding required in the Standards

Conceptual Understanding Examples

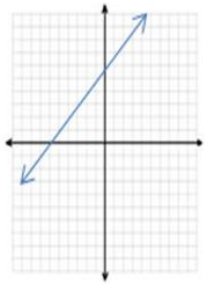
4.NF.C Understand decimal notation for fractions, and compare decimal fractions.

1.7 or 17 twelfths

a. Which number is larger?

b. Explain how you can tell without drawing a picture.

8.EE.C.8.a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.



a. Draw a line that intersects this line.

b. Write a system of equations that is represented by these two lines.

c. What is the solution to the system?

Procedural Skill and Fluency

Procedural Skill and Fluency

- The Standards require speed and accuracy in calculation.
- Materials structure class time and/or homework time for students to practice core functions so that they are more able to understand and manipulate more complex concepts.
- The high school standards do not set explicit expectations for fluency, but fluency helps students get past the need to manage computational details so that they can observe structure and patterns in problems.

Required Fluencies in K-6

Grade	Standard	Required Fluency
K	K.OA.5	Add/subtract within 5
1	1.OA.6	Add/subtract within 10
2	2.OA.2 2.NBT.5	Add/subtract within 20 (know single-digit sums from memory) Add/subtract within 100
3	3.OA.7 3.NBT.2	Multiply/divide within 100 (know single-digit products from memory) Add/subtract within 1000
4	4.NBT.4	Add/subtract within 1,000,000
5	5.NBT.5	Multi-digit multiplication
6	6.NS.2,3	Multi-digit division Multi-digit decimal operations

Procedural Skill and Fluency

5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm

1. $49 \times 975 =$ _____

2.
$$\begin{array}{r} 6{,}751 \\ \times 609 \\ \hline \end{array}$$

3. What is the product of 9 and 740?

4. Multiply 28 and 5,555.

Fluency in the Middle Grades

- Standards no longer use the key words of fluent or fluently
- Middle school fluency less about calculation
- Fluency at the middle level is about ease of manipulation
  - Expressions
  - Equations
  - Notations

Procedural Skill and Fluency

8.EE.C.7: Solve linear equations in one variable.

Decide whether the solution to each equation is positive, negative, zero, or there are no solutions. Check the correct box for each row.

	solution is positive	solution is negative	solution is zero	there are no solutions
$3x = 5$				
$5z + 7 = 3$				
$7 - 5w = 3$				
$4a = 9a$				
$y = y + 1$				

Developing Fluency

What

- More than just timed tests
- Deliberate selection of problems
- Different forms of repeated practice

Why

- Build automaticity

How

- Classroom instruction must emphasize the development and application of strategies
- Distributed over time, rather than in a single sitting

Application

Application

- Students can use appropriate concepts and procedures for application even when not prompted to do so
- Provide opportunities at all grade levels for students to apply math concepts in “real world” situations, recognizing this means different things in K-2, 3-5, 6-8, and high school
- Teachers in content areas outside of math ensure that students are using grade-level-appropriate math to make meaning of and access content

Problems vs. Exercises

Problems

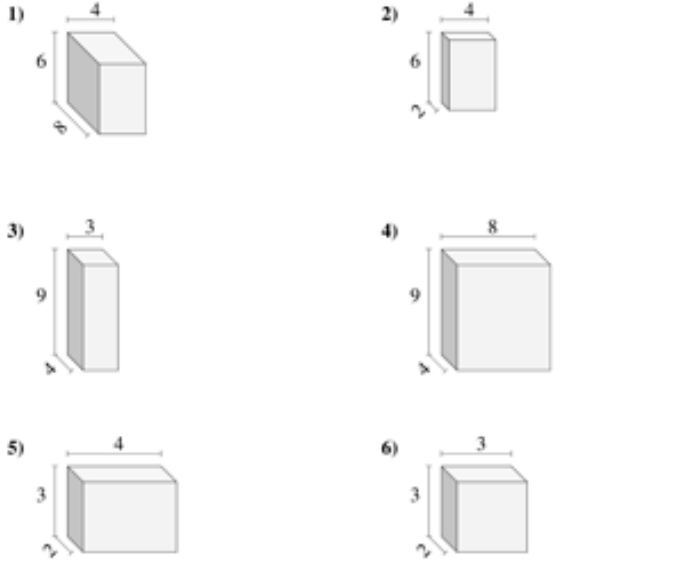
- Students learn new mathematics
- Students are asked to use what they already know to answer mathematical questions that they haven't been taught to solve
- Opportunity for discussion and allows teachers to see what students understand

Exercises

- Students apply what they have already learned to build mastery
- Repetition to develop a skill

Engaging With Problem Solving

Find the volume of each of the rectangular prisms. Measured in cm (not to scale).



7. A plastic rectangular container measures 5 inches wide, 8 inches long and 3 inches high. How much water will fit in the container?

8. In order to ship an item, the company requires a box that is 48 cubic feet. The box they currently have is 3 feet wide, 4 feet high and 2 feet long. Will this satisfy the companies requirements?

www.CommonCoreSheets.com

My pool is a rectangular shaped pool. It has a width of 10 feet and a length of 18 feet. The whole pool has a depth of 5.5 feet. The delivery trucks can carry 3,000 gallons of water and there are approximately 7.5 cubic feet in a gallon. They charge $0.04 per gallon and an extra delivery fee of $15 per truck load.

How much water is needed to fill the pool?

How many trucks are needed?

How much will the delivery cost?

Rigor in the Standards

The language of the standards assist in determining the appropriate use of rigor in instruction.

- Conceptual Understanding
- Procedural Skill and Fluency
- Application

Conceptual Understanding

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

x	-3	2	3
y	-3	7	9

A

x	0	2	4
y	5	7	9

B

x	-1	0	2
y	5	1	7

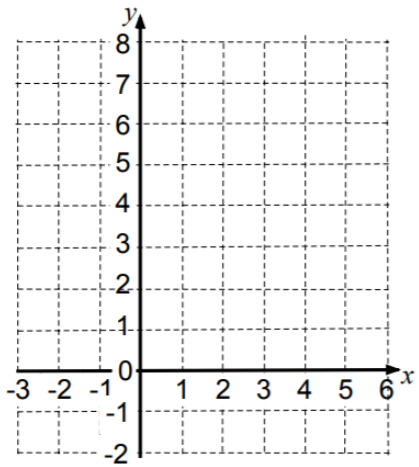
C

x	-1	0	2
y	1	3	7

D

1a. Which of these tables of values satisfy the equation $y = 2x + 3$? Explain how you checked.

b. By completing the table of values, draw the lines $y = 2x + 3$ and $x = 1 - 2y$ on the grid.



$y = 2x + 3$

x	-2	0	
y			5

$x = 1 - 2y$

x	0		5
y		0	

c. Do the equations $y = 2x + 3$ and $x = 1 - 2y$ have one common solution, no common solutions, or infinitely many common solutions? Explain how you know.

Procedural Skill and Fluency

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

Solve each of the following systems:

$-4x - 2y = -12$
$4x + 8y = -24$

$x - y = 11$
$2x + y = 19$

$8x + y = -1$
$-3x + y = -5$

$5x + y = 9$
$10x - 7y = -18$

Source: Leinwand, S., Brahier, D., and Huinker, D., Principles to Action, pg. 20
A.REI.C.6

Application

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

You are trying to decide which two smartphone plans would be better. Plan A charges a basic fee of $30 per month and 10 cents per text message. Plan B charges a basic fee of $50 per month and 5 cents per text message.

a) How many text messages would you need to send per month for Plan B to be the better option? Explain your decision?

b) If the cell phone company decided to offer unlimited texts for $80 per month, do you think that you would change your smartphone plan? Use mathematical reasoning to support your decision.

Adapted from Illustrative Mathematics and Leinwand, S., Brahier, D., and Huinker, D., Principles to Action, 2014, pg. 20
A.REI.C.6

What does student learning look like if the teacher is balancing rigor?

Look Fors

Students are either working *toward* or *using* understanding.

Conceptual Development

- Students work on tasks that elicit ideas, discussions, and connections that lead to new understanding
- Students ask and answer questions about understanding

Fluency and Procedural Skills

- Students engage in meaningful practice
- Students connect procedural skills with representations

Application

- Students apply knowledge to new, but accessible situations that extend their understanding

Rigor in Tasks

Task selection in critical in order to effectively address rigor in instruction.

- Conceptual Understanding
- Procedural Skill and Fluency
- Application

Summary of Rigor

- Balance of conceptual understanding, procedural skill and fluency, and application
- Targeted aspect of rigor in a lesson should be determined by the standard being addressed
- Equal intensity throughout the course of a year, not necessarily in each lesson or unit

Finding Evidence in the Lessons

Planning for Standards-Aligned Instruction

- Analyze and interpret the lesson plan to collect and discuss evidence of the shifts
- Review the guiding questions related to each shift in the Lesson Plan Analysis Template
- Respond to the questions using evidence from the lesson plan provided.

Conclusion

How do I know if the lesson reflects the Shifts?

- Is the lesson addressing on grade-level content?
- What is the full intent of the standard(s) being addressed?
- Is the aspect of rigor required by the standard(s) the same as the aspect(s) being addressed in the lesson?
- How does the lesson connect to and build on students' prior skills and knowledge?

Resources to Support Teachers

- Self-Paced Learning Modules
  - Math Module 1: Focus on Practice Standards
  - Math Module 2: Focus on Content Standard
  - Supporting Success for ALL Students with the CT Core Standards
  - CCS Mathematics Success for Students with Disabilities
  - CCS Mathematics Success for English Learners
- Illustrative Mathematics
  - Course Blueprints
  - Tasks and other resources for each content standard
- Student Achievement Partners
  - Major work of the grade
  - Textbook alignment and adaptations (enVisionmath 2.0, GO Math!)
  - Instructional Materials Evaluation Tool
  - Most Misunderstood Middle School Mathematics Standards
- YouCubed
  - Tasks
  - Research evidence on best ways to learn math facts

Linking Content Standards and Assessment

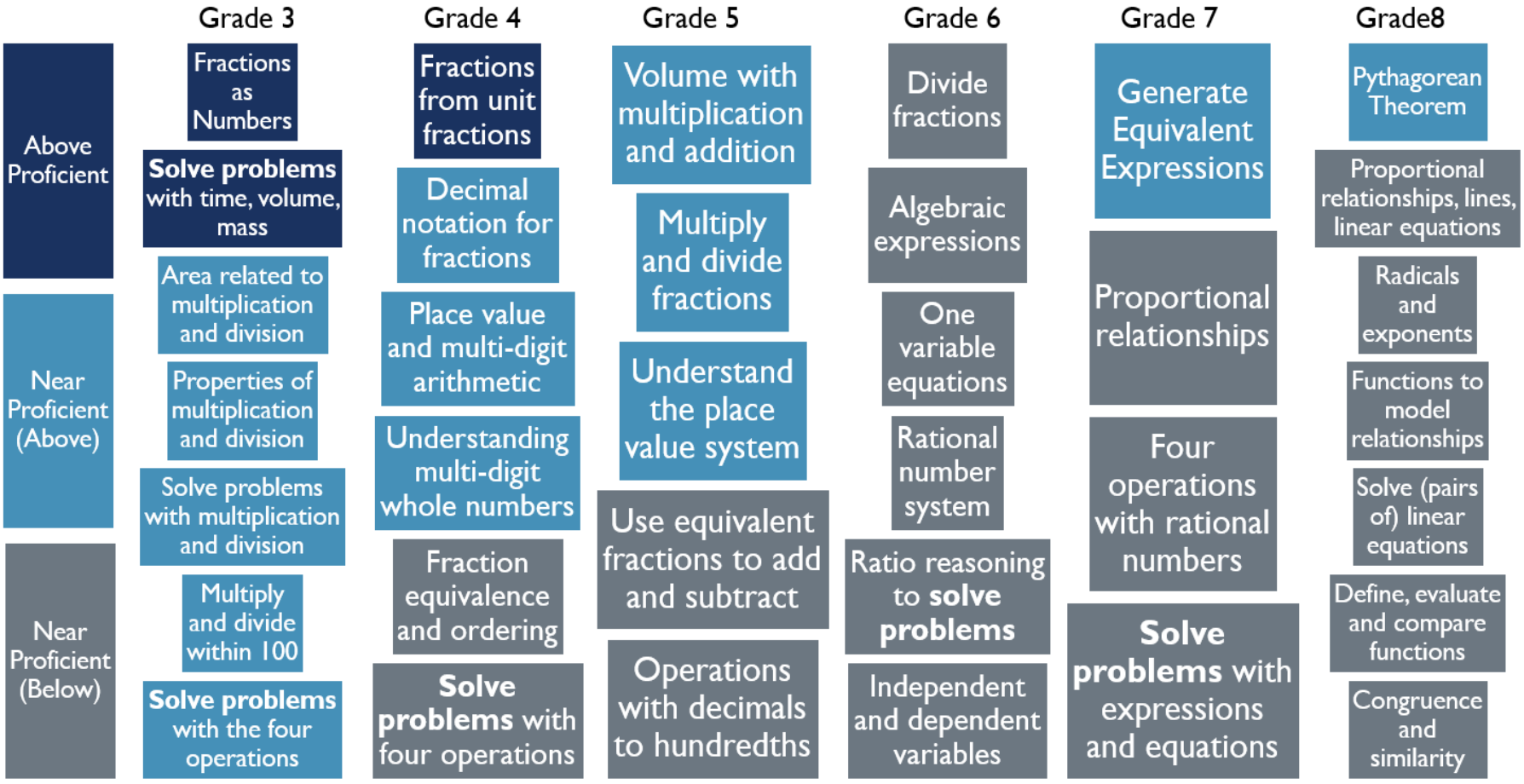
Claim 1

- Content standard clusters are the same as the targets
- Coherence within and across grades
- Logical pre-requisites needed for student success of mathematical content
- Mapping clusters to targets to understand connections between targets
- Each target has a specified DOK

Analyzing Targets to Inform Instruction

- Re-Analyzing Smarter Balanced Mathematics Target Results to Inform Instructional Improvement
  - By 2019 all grade three targets near proficiency
  - Targets related to problem solving have the greatest need
  - Fractions domain has steepest decline in cohort data
  - Younger students consistently higher performing than older peers
  - Depressed performance 6-8
  - Large differences between racial and ethnic groups and these differences grow over time
  - Large disparity between students who are economically disadvantaged and those that are not
  - Starkest difference between students identified as limited English proficiency
  - Targets with higher DOK expectations have lower performance

Major work of the Grade



Impact of Connections

M	+	3D	Solve problems involving the four operations, and identify and explain patterns in arithmetic.	4A	4C		
A	+	3E	Use place value understanding and properties of operations to perform multi-digit arithmetic.	4D	4E		
M	=	3F	Develop understanding of fractions as numbers.	3K	5F	4G	5J
M	+	3G	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and	4I			
S	-	3H	Represent and interpret data.	4J			
M	-	3I	Geometric measurement: understand concepts of area and relate area to multiplication and to	3J	5I	4I	5F
A	-	3J	Geometric measurement: recognize perimeter as an attribute of plane figures and 3-4 distinguish	4I			
S	+	3K	Reason with shapes and their attributes	4L	5K		
M	-	4A	Use The Four Operations With Whole Numbers To	5F	5A	6A	8B
A	+	4C	Generate And Analyze Patterns	5B			
S	+	4B	Gain Familiarity With Factors And Multiples	6E	6C		
M	+	4D	Generalize Place Value Understanding For Multi-Digit Whole Numbers	4E	5C	4A	
M	+	4E	Use Place Value Understanding And Properties Of Operations To Perform Multi-Digit Arithmetic	5D	4A		
M	+	4F	Extend Understanding Of Fraction Equivalence And	4H	5E	5F	4G
M	+	4G	Build Fractions From Unit Fractions By Applying And Extending Previous Understandings Of	4H	5E	5F	4I
M	+	4H	Understand Decimal Notation For Fractions, And Compare Decimal Fractions	4I	5C		
S	+	4I	Solve Problems Involving Measurement And Conversion Of Measurements From A Larger Unit	6H	5I	5F	5G
S	+	4J	Represent And Interpret Data.	5H			
A	+	4K	Geometric Measurement: Understand Concepts Of Angle And Measure Angles.	7F.5	HS.G-CO.A		
A	+	4L	Draw And Identify Lines And Angles, And Classify Shapes By Properties Of Their Lines And Angles.	5K			
A	+	5A	Write and interpret numerical expressions.	6E	6C		
A	-	5B	Analyze patterns and relationships.	6E	6G	6F	6A

Resources to Support Teachers

- Content Explorer
  - Learn about test development
    - Blueprints
    - Scoring rubrics
  - Support Instruction
    - Plan activities aligned to grade-level claims, targets, and standards
    - Understand how standards related to claims and targets
      - How are academic standards assessed?

- Interim Assessments
  - Three types of blocks
    - Interim Comprehensive
    - Interim Assessment Blocks
    - Focused Interim Assessment Blocks
  - Fixed form tests
    - Some overlap of test items on the ICA and blocks
  - Blocks is associated with a Connections Playlist in the Digital Library


QUESTIONS

Thank You

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