CONNECTICUT MASTERY TEST

FOURTH GENERATION

MATHEMATICS HANDBOOK

Connecticut State Department of Education 2006

CONNECTICUT STATE DEPARTMENT OF EDUCATION

Betty J. Sternberg, Commissioner

Division of Teaching, Learning and Assessment

Frances Rabinowitz
Associate Commissioner

Bureau of Research, Evaluation and Student Assessment

Barbara Q. Beaudin, Chief

William Congero, Director of Student Assessment
Steve Martin, CMT Coordinator
Abe Krisst, CMT Mathematics
Gail Pagano, CMT Mathematics

Bureau of Curriculum and Instruction

Barbara Westwater, Chief

Charlene Tate-Nichols, Mathematics Consultant

Office of Communications

Donald G. Goranson, Jr., Editor
Janet I. Montague, Desktop Publisher
Andrea Wadowski, Graphic Designer

Connecticut Mastery Test
Fourth Generation
Mathematics Handbook

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FOREWORD

The *Connecticut Mastery Test Fourth Generation Mathematics Handbook* is intended to provide teachers and other educators with important information about the mathematics subtests of the fourth generation of Mastery Tests that will be launched in the spring of 2006. This handbook is provided electronically and will be updated to include additional activities, instructional strategies and scored student work.

The materials contained in this handbook answer many of the questions Connecticut's educators have asked about the tests. Accordingly, one will find here:

- a summary of the changes made in the content to be assessed and in test formats;
- detailed test blueprints, including the number and types of items and the specific skills and concepts that will be assessed at Grades 3, 4, 5, 6, 7 and 8;
- sample items for each strand to be assessed at Grades 3 through 8;
- generic scoring rubrics for open-ended items;
- grade-level mathematics vocabulary lists;
- teaching suggestions and activities, including web links; and
- scored student work for open-ended items from spring test administration pilots at all six grades.

It is hoped that the materials in this handbook will help to continue the ongoing process of improving mathematics instruction in Connecticut's public schools.

Dr. Betty J. Sternberg
Commissioner of Education

ACKNOWLEDGMENTS

The Connecticut State Department of Education wishes to thank the following educators, who were members of the external review committee for this publication. Their expertise and efforts in developing the *CMT Fourth Generation Mathematics Handbook* were invaluable.

Tina Della Bernarda, Bristol Public Schools
Betsy Carter, Hamden Public Schools
Sandra Coelho, PIMMS, Wesleyan University
Gail Pagano, Cromwell Public Schools
Phillip Gedeon, Connecticut College
Lauren Law, Putnam Public Schools
Marie Mas, Regional School District 9
Pedro Vasquez, Bridgeport Public Schools
Janice Vuolo, mathematics consultant
Michelle Watson, Wallingford Public Schools

INTRODUCTION

The Fourth Generation of the Connecticut Mastery Test (CMT) in mathematics assesses student performance on a range of skills and concepts expected to be mastered by students in Grades 3, 4, 5, 6, 7 and 8. Connecticut's Mastery Test Program will be expanded in 2005-2006 to include Grades 3, 5 and 7, as mandated by the federal No Child Left Behind (NCLB) legislation. The skills and concepts are representative of and aligned with the content and performance standards in Connecticut's Mathematics Curriculum Framework. The framework includes mathematics content and instructional processes which are recommended by the National Council of Teachers of Mathematics (NCTM) and assessed on the National Assessment of Educational Progress (NAEP).

The *Connecticut Mastery Test Fourth Generation Mathematics Handbook* is designed to ensure closer links between what and how mathematics is taught in Connecticut schools, and how mathematics is assessed on the CMT. In addition, this handbook has been designed to ensure that all Connecticut educators – particularly teachers of mathematics in Grades K through 8 are familiar with the content, format and level of mastery expected on the mathematics CMT.

The CMT Mathematics Handbook is intended to be a resource for teachers as they develop meaningful mathematics programs for students in Grades K-8 that are aligned with the spirit and content of Connecticut's Mathematics Curriculum Framework as reflected by the CMT. This handbook provides.

- a summary of the changes incorporated into the fourth generation tests;
- test blueprints and content outlines;
- rubrics for scoring open-ended items;
- sample items for each strand to be assessed at Grades 3 through 8;
- instructional strategies and activities linked to content standards at each grade level;
- mathematics vocabulary, by grade level, with which all students should be familiar to be successful in mathematics; and
- scored student work for open-ended items from spring test administration pilots at all six grade levels.

Overall, the mathematics content of the CMT should be viewed as one component of a comprehensive, standards-based, mathematics program designed to set and meet high expectations for all students. While the CMT mathematics objectives should not be viewed as a curriculum, all district-level mathematics curriculums should include the skills and concepts assessed on the CMT. Daily classroom instruction should not be limited to preparation for the CMT. However, high-quality instruction should naturally reflect what is assessed and how it is assessed on the CMT.

It is hoped that the content of this handbook will provide teachers of mathematics with the information and ideas they need to continue to build and implement high-quality programs that significantly improve the mathematical literacy of all Connecticut students.

Position Statement on Mathematics Education

Adopted June 21, 2000, by the Connecticut State Board of Education

As part of everyone's daily routine, we are regularly presented with problems that require us to accurately compute sums, differences, products and quotients, analyze data, make predictions, recognize patterns and draw conclusions. In all of these instances, the abilities to compute accurately and to make reasonable estimates are required. A strong mathematics program provides for a comprehensive and sequential approach in which the acquisition of basic skills (addition, subtraction, multiplication and division) serves as the foundation for more complex problem solving and conceptual understanding.

The State Board of Education believes that the recent debate pitting the acquisition of basic skills against the development of conceptual understanding argues a false dichotomy. Rather, basic skills and conceptual understanding are intertwined, and both are necessary before students can successfully apply mathematics to the solution of problems. A strong mathematics program will enable students to do each with ease.

Unfortunately, not enough students in Connecticut or in the nation are sufficiently developing the facility, understanding, level of confidence and interest in mathematics to meet our present and future societal needs. Therefore, we must fully engage in the quest to provide every student with a strong mathematics program, beginning in the earliest grades.

Accordingly, the Connecticut State Board of Education believes that every student needs and deserves a high-quality, comprehensive mathematics education program that develops mathematical facility in the basic skills and quantitative literacy in numbers, measurement, algebra, geometry and statistics. To meet this goal and to best serve Connecticut's students, we encourage educators to adopt the following measures:

- overall, set higher expectations for **all** students to ensure earlier and more equitable opportunity to learn mathematics;
- in curriculum, provide a more rigorous study of mathematical skills and concepts and their applications in today's world for both career and personal decisions, and a more coherent and coordinated pre-K-12 program of instruction;
- in teaching, create classrooms that are stimulating learning environments in which **all** students have the opportunity to reach their full mathematical potential and in which, working collaboratively with families, **all** students are inspired to do so;
- in learning, provide more active student involvement with mathematics, including mathematical problems that relate to their present world and their future career needs and demands, and the use of a variety of mathematical tools for solving those problems;
- in technology, foster more systematic and appropriate use of technological tools to enhance access to mathematics concepts;

- in professional development, provide more professional collaboration and teacher externships to provide a stronger focus on the underlying mathematics being taught; and

- in assessment, provide student evaluations that are continuous and based on many sources of evidence.

These measures, embodied in the goals and standards outlined in Connecticut's *Guide to K-12 Program Development in Mathematics* and in the *Connecticut Framework: Preschool Curricular Goals and Benchmarks*, should result in more mathematically powerful students who demonstrate the ability to:

- compute (using addition, subtraction, multiplication and division, when appropriate, with whole numbers, fractions, decimals and percentages) and use mathematical concepts and skills to make and justify decisions and predictions, to identify patterns and trends, to pose questions from data and situations, and to formulate and solve problems;

- select and use appropriate approaches and tools for solving computational, geometrical and algebraic problems, including estimation, mental computation, paper and pencil, manipulative materials, calculators, and computers with software for tabulating, charting, graphing, drawing, and transforming data and images;

- use mathematical skills and concepts to describe and analyze data and measurements of physical and social phenomena from other disciplines;

- communicate numerical, geometrical, algebraic and statistical ideas orally and in written forms with models, pictures, graphs and mathematical symbols, using paper and pencil, a variety of calculator displays, spreadsheets, graphing packages, word processing and other related computer software;

- use inductive and deductive reasoning to make, defend and evaluate conjectures and arguments, to justify assertions and verify tentative conclusions, and to solve mathematical problems; and

- identify and use connections within mathematics to identify interrelationships and equivalent representations, to construct mathematical models, and to investigate and appreciate mathematical structure.

We take these positions to ensure that all students, by the end of Grade 12, will apply proficiently a range of numerical, algebraic, geometric and statistical concepts and skills to formulate, analyze and solve real-world problems; to facilitate inquiry and the exploration of real-world phenomena; and to support continued development and appreciation of mathematics as a discipline.

SUMMARY OF CHANGES FROM THIRD GENERATION CMT TO FOURTH GENERATION CMT

Test

- A test is mandated for each grade, 3-8, inclusive.
- The test will be administered in the spring of the school year, beginning with spring 2006.
- There are no separate answer booklets. Students are expected to record their responses in the spaces provided in test booklets. Students will be permitted to underline, highlight, and make marks in their test booklets.
- Three separate test booklets are provided, one each for mathematics, reading and writing.
- There are no grid-in items for Grades 3 and 4.
- Grade-appropriate rulers are provided (see page 2).
- A formula sheet is provided for Grade 8 (see pages 3 and 330).
- The total number and percentage of multiple-choice items decreases while the total number and percentage of constructed-response items increases as the grade level increases.

Reporting Format

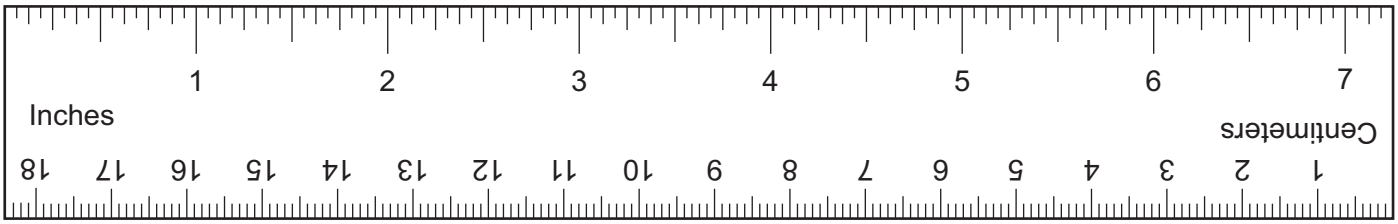
- The same 25 content strands are tested. The strands will be organized by the four standards outlined in the Mathematics Curriculum Framework, including:
    - numerical and proportional reasoning;
    - geometry and measurement;
    - working with data: probability and statistics; and
    - algebraic reasoning: patterns and functions.

The CMT scores will continue to be reported by strand.

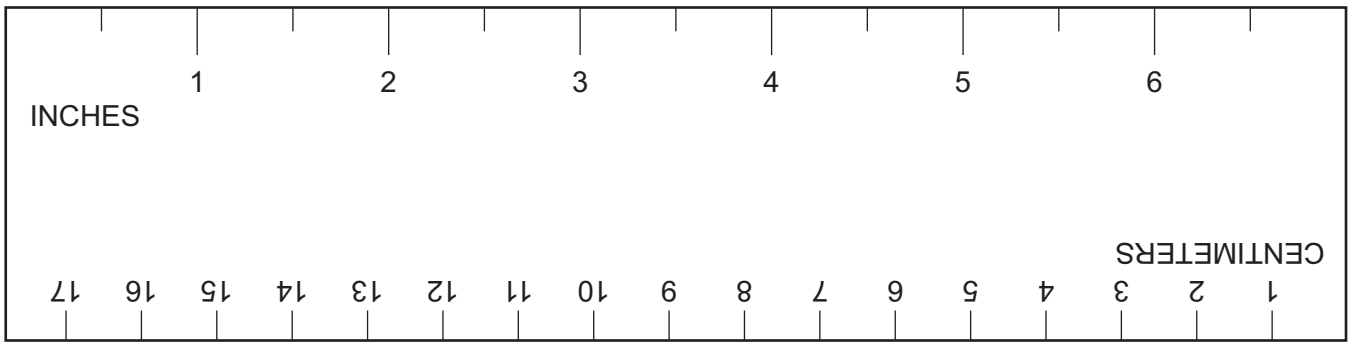
Handbook

- This document will be available on the CSDE website, on CD and in limited quantity in hard copy.
- Information is organized by grade level.
- Vocabulary lists are cumulative. New vocabulary at a grade level is in bold print.

Rulers For Use During The Connecticut Mastery Test

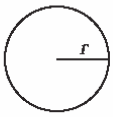
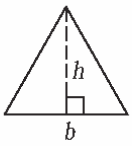
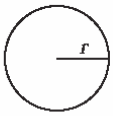
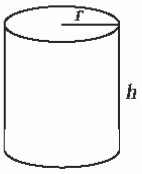
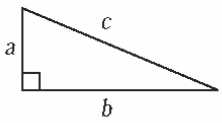


Grades 5, 6, 7 & 8



Grades 3 & 4

CMT Formula Chart

Circumference	circle	$C = 2\pi r$	
	pi = π	Use 3.14 OR $\frac{22}{7}$	
Area	triangle	$A = \frac{1}{2}bh$ 	circle $A = \pi r^2$ 
Volume	cylinder	$V = \pi r^2 h$	
Pythagorean Theorem	right triangle	$a^2 + b^2 = c^2$	

Measurement Conversion

Customary Length	1 mile = 5,280 feet
Customary Volume	1 gallon = 4 quarts 1 quart = 2 pints 1 pint = 2 cups 1 cup = 8 ounces
Customary Weight	1 ton = 2,000 pounds 1 pound = 16 ounces
Time	1 year = 365 days 1 year = 52 weeks

ORGANIZATION OF CMT STRANDS BY STANDARD

NUMERICAL and PROPORTIONAL REASONING
1. Place Value
2. Pictorial Representations of Numbers
3. Equivalent Fractions, Decimals and Percents
4. Order, Magnitude and Rounding of Numbers
5. Models for Operations
6. Basic Facts
7. Computation with Whole Numbers and Decimals
8. Computation with Fractions and Integers
9. Solve Word Problems
10. Numerical Estimation Strategies
11. Estimating Solutions to Problems
12. Ratios and Proportions
13. Computation with Percents
GEOMETRY and MEASUREMENT
14. Time
15. Approximating Measures
16. Customary and Metric Measures
17. Geometric Shapes and Properties
18. Spatial Relationships
WORKING with DATA: PROBABILITY and STATISTICS
19. Tables, Graphs and Charts
20. Statistics and Data Analysis
21. Probability
24. Classification and Logical Reasoning
ALGEBRAIC REASONING: PATTERNS and FUNCTIONS
22. Patterns
23. Algebraic Concepts
INTEGRATED UNDERSTANDINGS (May include content from one or more of the four Standards)
25. Mathematical Applications

SUMMARY OF CHANGES
FROM THIRD TO FOURTH GENERATION

CMT-3

Grade	4	6	8
# of Sessions	2	3	3
# of Reporting Strands	18	23	23
# of Items	94	116	121
Multiple-choice items	76	80	70
Open-ended Items	18	23	32
Grid-in Items	0	13	19

CMT-4

Grade	3	4	5	6	7	8
# of Sessions	2	2	3	3	3	3
# of Reporting Strands	18	21	23	23	23	21
# of Items	94	96	113	116	120	117
Multiple-choice items	76	80	80	71	70	61
Open-ended Items	18	16	20	27	31	36
Grid-in Items	None	None	13	18	19	20

CMT-4 Mastery Levels

Maximum Points In Strand	Points Needed for Mastery
4	3
6	4
8	6
9	6
10	7*
12	8*

***mastery level pending**

Calculators are NOT allowed for Grades 3 and 4. Calculators ARE allowed for Grades 5 through 8 in sessions 2 and 3.

Mastery Criteria Map for CMT Generation 4 Mathematics Strands

Standards and Content Strands	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Numerical and Proportional Reasoning						
1. Place Value	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**3 out of 4**
2. Pictorial Representations of Numbers	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**NT**
3. Equivalent Fractions, Decimals and Percents	**NT**	**3 out of 4**	**3 out of 4**	**3 out of 4**	**3 out of 4**	**4 out of 6**
4. Order, Magnitude and Rounding of Numbers	**4 out of 6**	**4 out of 6**	**6 out of 8**	**6 out of 8**	**6 out of 8**	**4 out of 6**
5. Models for Operations	**6 out of 8**	**6 out of 8**	**6 out of 8**	**4 out of 6**	**4 out of 6**	**4 out of 6**
6. Basic Facts	**4 out of 6**	**4 out of 6**	**4 out of 6**	**3 out of 4**	**NT**	**NT**
7. Computation with Whole Numbers and Decimals	**4 out of 6**	**4 out of 6**	**4 out of 6**	**6 out of 8**	**4 out of 6**	**4 out of 6**
8. Computation with Fractions and Integers	**NT**	**3 out of 4**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**
9. Solve Word Problems	**4 out of 6**	**3 out of 4**	**4 out of 6**	**6 out of 8**	**6 out of 8**	**6 out of 8**
10. Numerical Estimation Strategies	**3 out of 4**	**3 out of 4**	**4 out of 6**	**3 out of 4**	**3 out of 4**	**NT**
11. Estimating Solutions to Problems	**3 out of 4**	**3 out of 4**	**3 out of 4**	***7 out of 10***	***7 out of 10***	**6 out of 8**
12. Ratios and Proportions	**NT**	**NT**	**NT**	**3 out of 4**	**3 out of 4**	**6 out of 8**
13. Computation with Percents	**NT**	**NT**	**NT**	**NT**	**3 out of 4**	**4 out of 6**
Geometry and Measurement						
14. Time	**4 out of 6**	**3 out of 4**	**3 out of 4**	**NT**	**NT**	**NT**
15. Approximating Measures	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**
16. Customary and Metric Measures	**4 out of 6**	**3 out of 4**	**6 out of 8**	**6 out of 8**	**6 out of 8**	**6 out of 8**
17. Geometric Shapes and Properties	**4 out of 6**	**4 out of 6**	**4 out of 6**	**6 out of 8**	**6 out of 8**	**6 out of 8**
18. Spatial Relationships	**NT**	**NT**	**3 out of 4**	**4 out of 6**	**6 out of 8**	***8 out of 12***

Working with Data: Probability and Statistics						
19. Tables, Graphs and Charts	**6 out of 8**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**
20. Statistics and Data Analysis	**NT**	**NT**	**3 out of 4**	**3 out of 4**	**4 out of 6**	**6 out of 8**
21. Probability	**3 out of 4**	**3 out of 4**	**4 out of 6**	**3 out of 4**	**4 out of 6**	**4 out of 6**
24. Classification and Logical Reasoning	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**
Algebraic Reasoning: Patterns and Functions						
22. Patterns	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**
23. Algebraic Concepts	**NT**	**3 out of 4**	**3 out of 4**	**4 out of 6**	**6 out of 8**	***7 out of 10***
Integrated Understandings						
25. Mathematical Applications	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**	**4 out of 6**

NT: Strand not tested at this grade level
Italicized: Mastery criteria is pending

Point Values For Each Standard On The CMT Generation 4 Test

Standard	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Numerical and Proportional Reasoning	52	58	66	74	72	64
Geometry and Measurement	24	20	28	28	30	34
Working with Data: Probability and Statistics	18	16	22	20	24	26
Algebraic Reasoning: Patterns and Functions	6	10	10	12	14	16
Total Raw Points Minus Strand 25	100	104	126	134	140	140
Total Raw Points	106	110	132	140	146	146

Math Session Testing Times

	GRADE					
	3	4	5	6	7	8
Session 1	60 min	60 min	60 min	60 min	60 min	60 min
Session 2	60 min	60 min	60 min	60 min	60 min	60 min
Session 3	NA	NA	60 min	60 min	60 min	60 min