**Activity 8.5.4 Sierpinski Carpet**

In Activity 8.5.2, you used GeoGebra to construct several stages of a Sierpinski triangle. In this activity you will start with a square instead of a triangle. You will divide the square into smaller congruent squares and remove the interior of the middle square. Then you will repeat this process on squares that did not have their interiors removed. The result of applying this process over and over again is called a Sierpinski carpet. Just as you did for the Sierpinski triangle, you will calculate the perimeter and area at each stage of your construction.

1. Open GeoGebra. Close the algebra side panel and click on the axes icon to remove the axes.

2. Create Stage 0.

a. Use the regular polygon icon to construct a square. Make sure that you can move one of the vertices on your square to make it larger or smaller. Then enlarge the square so that it fills most of your screen and its base is parallel to the bottom of the screen. This is Stage 0 of the Sierpinski Carpet.

b. Assume that the side length of the square is 1 unit. What is the square’s perimeter? What is its area.?

3. Stage 1.

a. Create Stage 1 of a Sierpinski carpet.

* Create two points on the left side the square that divide this side into thirds. Then use the regular polygon tool to divide the Stage 0 square into 9 congruent squares.
* Right click on your square and select Object Properties.
* Select Poly 1. Click on a color of your choice and then slide the slider to darken the color of the original square.
* Point one at a time to the items in the Polygon list until you identify the square in the middle. Then click on this item and select white for the color. Slide the slider all the way to the right to make the interior of this square white, indicating that the interior of this square has been removed.
* Click on Segment to highlight all the segments and then select the color black.
* Close the Object Properties box.

b. Make a sketch of Stage 1.

c. The perimeter of Stage 1 is the perimeter of Stage 0 plus the perimeter of the center square (whose interior was removed). Find the perimeter of Stage 1.

d. The area of Stage 1 is the area of Stage 0 minus the area of the interior of the center square that was removed. Find the area of Stage 1.

4. Create a tool that can be used to create other stages. Here’s how:

* Click the Tools tab and select Create New Tool.
* Click the down arrow on the Select Objects bar and select everything that GeoGebra allows you to select.
* Click Next. Notice that the input is simply two points, Point A and Point B.
* Click Next. Name your tool “Carpet.” Write: “Click on two points” in the Tool Help window.

5. Stage 2.

a. For Stage 2, apply the tool that you created in question 4 to the 8 small squares whose interiors have not been removed. This application should divide each square into 9 congruent squares and then remove the interior of the center square. Make a sketch of Stage 2.

b. How much has the perimeter changed from Stage 1 to Stage 2? Find the perimeter of Stage 2.

c. How much has the area changed from Stage 1 to Stage 2? Find the area of Stage 2.

6. Stage 3.

a. For Stage 3, apply the tool that you created in question 4 to the small squares from Stage 2 whose interiors have not been removed. How many small squares are there?

b. How much has the perimeter changed from Stage 2 to Stage 3? Find the perimeter of Stage 3.

c. How much has the area changed from Stage 2 to Stage 3? Find the area of Stage 3.

7. Let *Pn* represent the perimeter and *An* the area of stage *n* of the Sierpinski carpet. Find recursive formulas that give *Pn* in terms of *Pn–1* and *An* in terms of *An–1*.

8. Mobile phone and WiFi fractal antennas have been produced in the form of a few stages of the Sierpinski carpet. They are easy to produce and smaller than other conventional antennas. See, for example, <http://www.ijsret.org/pdf/rahul_batra.pdf>. The fractal design for the antenna gives a lot of perimeter for a small amount of area. Show that as you progress from Stage 0 to Stage 3, the ratio of perimeter to surface area increases.