**Main Problem #1**

Topic: *Add Fractions with Unlike Denominators*

Problem: In upper- level mathematics there exists a geometric problem called the “Halfway Problem” in which your goal is to determine whether or not the sum of a number sequence equals 1. Every number after the first element in the sequence is half of the previous element.

To better explain this problem, let us imagine that you and your friend Eduardo are playing “Red Light Green Light.” The distance between you and Eduardo is 1 meter. The first time Eduardo says “Green Light”, you advance half of a meter. The second time he says “Green Light”, you advance a quarter of a meter. The third time he says it, you advance an eighth of a meter. The game goes on, with you advancing half of the preceding distance, until you reach the other side.

Q1. What is the sum of the first 5 distances you travel?

Q2. Model the first 5 distances within a Fraction Square.

Q3. How many “Green Lights” will it take for you to reach the other side? *Hint: Continue the sequence.*

Q4. Can we approximate that the distance traveled to be 1 meter?

A1. According to given information, the first three distances, in meters, are $\frac{1}{2}$, $\frac{1}{4}$, and$\frac{1}{8}$. Because every other distance is half of the preceding number (element), the fourth distance (element) is $\frac{1}{16}$and the fifth distance is $\frac{1}{32}$.

To find the sum, we must first find the least common multiple of all the denominators. Luckily, the LCM is 32. Therefore, the sum is: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}=\frac{16}{32}+\frac{8}{32}+\frac{4}{32}+\frac{2}{32}+\frac{1}{32}=\frac{31}{32}$.

A2. The Fraction Square below shows how the distances, in fractional representation, construct the area of a square with area 1.

1/2

1/4

1/8

1/16

1/32

A3. If we continue the sequence from where we left off ($\frac{31}{32}$), we would notice that the sum of *n* elements would not equal 1, rather its value would be really close to it. The example below shows the sum of the next 3 elements in this sequence.

LCM = 254

$$\frac{31}{32}+\frac{1}{64}+\frac{1}{128}+\frac{1}{256}=\frac{248}{256}+\frac{4}{256}+\frac{2}{256}+\frac{1}{256}=\frac{255}{256}$$

Therefore, it would take you infinite “Green Lights” until you reach 1 meter. Fun Fact: This is an example of an *infinite sequence*.

A4. Yes, we can approximate the distance to be 1 meter. In fact, the value of infinite turns (“Green Lights” is approximately 0.9999...It is perfectly valid for one to approximate the value to be 1.