**Unit 1: Investigation 2 (2 Days)**

**Relations and Functions**

**Common Core State Standards**

• F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then *f*(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = *f(*x).

• F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

• F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the* *function h(n) gives the number of person-hours it* *takes to assemble n engines in a factory, then the* *positive integers would be an appropriate* *domain for the function*.

**Overview**

This investigation is a review of functions from Algebra 1. Students review the difference between relations and functions, the domain and range of a function, and 1-1 functions. This material should be familiar to students who have completed Unit 3 of the Algebra 1 curriculum. In Algebra 2, there will be greater emphasis on asking students to translate between various representations of functions, and to investigate relations and functions in realistic contexts. Students will also connect their understanding of functions to Geometry by considering if the conic sections can be considered functions or not.

**Assessment Activities**

**Evidence of success: What will students be able to do?**

* Be able to determine if a given relation is a function or not.
* Given a function, be able to determine its domain and range.
* Be able to represent a function by an equation, table, graph, or verbal description and move comfortably from one representation to another.
* Determine the reasonableness of the domain and range of a function in a realistic context.
* Determine whether a function is a 1-1 function from a graph or table of values.
* Determine whether the basic conic sections are functions or not.

**Assessment Strategies: How will they show what they know?**

* **Exit Slip 1.2** gives students a set of relations in the form of an equation, table, or graph. For each of these, students determine whether the relation is a function. If so, students provide the other three representations for the function, and determine if the function is 1-1 or not.
* **Journal Entry 1** asks students to describe a realistic situation that can be modeled by a function, and asks them to provide a reasonable domain and range for the function.
* **Activity 1.2.1 Function Review** includes concepts covered in Unit 3 of algebra 1, including relations and functions, domain and range of a function, and different representations of functions (as an equation, table, graph, or verbal description).
* **Activity 1.2.2 What’s Reasonable?** asks students to consider a wide range of realistic situations; for each, students must decide if the situation could be described by a function, and if so, to find a reasonable domain and range for the function.
* **Activity 1.2.3a Function Junction** provides relations represented by either a table or graph. For each, students must determine whether or not the relation is a function. In **Activity 1.2.3b Is the Function 1-1?**, students determine whether the functions from Activity 1.2.3a are 1-1 or not.
* **Activity 1.2.4a Are Conics Functions? (GeoGebra version) and Activity 1.2.4b Are Conics Functions? (non-GeoGebra version)** ask students to determine if the conic sections (circle, ellipse, parabola, and hyperbola) are always a function, sometimes a function, or never a function.

**Launch Notes**

Ask the class the following set of questions:

* How many of you work a job for pay outside of school?
* For those of you who work, how many hours per week do you work?
* How is your paycheck calculated?
* Suppose that when you start a new job, your boss told you “You are working 10 hours every week for me. Some weeks I will pay you $87.00 for 10 hours of work, and other weeks I will pay you $101.00 for 10 hours of work. You won’t know which it will be until you actually get your paycheck.” How would you react? (These figures are based on a 10-hour work week; as of January 1, 2014, the minimum wage in Connecticut is $8.70 per hour, scheduled to increase to $10.10 per hour by January 2017.)

Why can’t this last situation occur? Because if we know the amount of time someone works, we know the amount of pay—in other words, if we know the input (number of hours), we know the exact value of the output (amount of pay). This is a simple example of a function, where if we are given a value for an input, there is only one possible value for the output.

**Teaching Strategies**

You may wish to use Activity 3.1.2 from Algebra 1 as a formative assessment before deciding how to use **Activity 1.2.1 Function Review**.

**Pair Activity – Activity 1.2.1 – Function Review**

Ask students individually to get out a piece of paper and to write down their best answers to the questions “In mathematics, what is a relation?” and then “In mathematics, what is a function?” Ask students to form pairs or small groups and to combine their responses into one set of answers to the questions.

Call on pairs/groups of students one at a time to write the essential parts of their responses on the board, first for relations and then for functions. When each pair/group of students has shared their responses, ask the whole class if there are any other ways to describe relations/functions that do not yet appear on the board. Provide a summary of students’ responses on the board for relations, then for functions. If the following do not appear in students’ responses, be sure to include them in the summary:

* A *relation* is a mapping from one set to another set. The elements of the first set form the domain of the relation, and the elements of the second set form the range of the relation.
* A *function* is a particular type of a relation, where each element in the domain is associated with exactly one element in the range. We can think of the domain as the set of inputs into the function, and the range as the set of outputs of the function. A function can be represented in a variety of ways: as a mapping diagram, a table, a set of ordered pairs, an equation, or a verbal description.

Based on students’ responses and/or the results of your formative assessment, Activity 1.2.1might be reviewed in class, in full or in part. Alternatively, Activity 1.2.1 could be given as homework.

**Pair Activity – Activity 1.2.1 – Function Review**

If you use Activity 3.1.2 from Algebra 1 as a formative assessment, consider using Activity 1.2.1 as an in-class activity, pairing up a stronger student with a weaker student. Tell the class that you might call on *any* student for answers to the questions, so that all of them are responsible for understanding the material. This is an excellent way for stronger students to deepen their own understanding, by helping to explain to another student.

In Activity 1.2.1 and throughout Algebra 2, students will be asked to consider the domain and range of a function. Question #3 of Activity 1.2.1 asks students to find the natural domain of a function. Be sure to remind students that the natural domain of a function is the largest set of values for which the function is defined. The domain is the set of inputs of a function, and may or may not be the same as the natural domain. For example, if we consider the function f(x) = x2, the natural domain of f is the set of all real numbers, but we might want to consider its domain to be the set of all nonnegative real numbers if we want to find the inverse of f. Also note the directions for questions #6 and #7 on creating a graph from a table of values. Be sure to point out to students that some functions have a discrete domain that consists only of a finite set of points, whereas others have a continuous domain that consists of all the points in an interval. (The same kind of direction is given in Activity 1.2.3a.)

Activity 1.2.1 includes an example using the GeoGebra software package. GeoGebra is available as a free download from <https://www.geogebra.org/>. This Activity provides an excellent opportunity to introduce students to this software if they have not used it before. If students are not familiar with GeoGebra, consider doing this part of Activity 1.2.1 (Question #3) together as a group.

Activity 1.2.1 also includes a question (#6) that depends on students’ understanding exponential decay or depreciation (a car is depreciating by 15% per year). This topic is covered in Algebra 1; see especially Activities 7.5.1 and 7.5.2 of Investigation 5 Unit 7 and. Exponential functions are also needed in Activity 3.6.3 of Unit 3 where polynomial and exponential growth are compared, and Activity 4.3.5 of unit 4 of the Algebra 2 curriculum. Material from Algebra 1 unit 7 will be needed in order to complete the study of the exponential family in Unit 5 of the Algebra 2 curriculum. Question #6 should be included if students have seen the relevant material before, but it could be skipped if it is thought that this material would be too much of a diversion at this point. References to the exponential functions continue in this unit as well as units 3 and 4 so the relevant activities from unit 7 Algebra 1 should be completed now if at all possible or reviewed if students had unit 7 and need a review.

**Differentiated Instruction (Enrichment)**

As an alternative or a supplement to Activity 1.2.1, consider asking students to create their own questions about functions—for example, ask students to create a table of values and have them determine if their tables could represent functions.

Introduce **Activity 1.2.2 What’s Reasonable?** to the class through a scenario such as the following: The Math Club at school is selling pies on Pi Day (3-14, or March 14) as a fundraiser. They will sell individual pieces of pie or a whole pie for a set price. How much money will they raise? Ask the class to determine what the variables should be, then if the variables would have a functional relationship and why. Now ask each student to write down the maximum number of pies they think the Math Club might sell. Then have students share their answers. Point out there is not a “correct” answer, but there is probably a range of reasonable answers. Now ask students to say what would be a reasonable domain and a reasonable range for the Pie Sales Function. Students should realize that a reasonable domain and range for such a function is the set of possible amounts of pie sold and the set of possible amounts of money the pie would sell for.

**Journal** **Entry** **1** Think about some mathematical situation you have encountered recently that might be described by a function. Describe the situation, and determine the equation of a function that might describe that situation. Be sure to make clear the meaning of the variables in your equation, and to use function notation. What do you think would be a reasonable domain and range of your function?

[Possible answers to this question: A Student might respond by giving an example of a recent purchase or a monthly payment, such as a fitness center or cell phone plan; or their income as a function of the number of hours worked. There is a very wide range of possible answers to this question; you should be sure that their response makes clear the meaning of the variables and that the domain and range they give are reasonable. (For example, it is unlikely an Algebra 2 student would work more than 30-40 hours in a week.)]

To introduce **Activity 1.2.3**, show examples of the graphs of two functions to the class, a quadratic function and an exponential function (without naming the functions at this point—you might simply say something like “we will study both of these kinds of functions in Algebra 2 and you may have seen them in Algebra 1”). Ask students how they are similar and how they are different. Point out that students have probably heard the expression “What goes up must come down.” Functions like the quadratic function are often used to model the height of something like a ball thrown into the air as a function of the amount of time since it was thrown. Notice that the object will be at the same height twice—once when it is on the way up and once when it is on the way down. On the other hand, functions like the exponential function are often used to describe how something grows over a period of time. If something is growing all the time, it won’t have the same size twice. The mathematical way to describe the difference between the functions is that the quadratic function is not 1-1, but the exponential function is.

**Activity 1.2.3a Function Junction** asks students to determine if a graph or table represents a function; if so, they are to represent the function in the “other” way (as a table if given as a graph, or as a graph if given as a table). **Activity 1.2.3b Is the Function 1-1?** asks students to consider whether the functions from 1.2.3a are 1-1 or not. These activities can be done in class (preferably in pairs) or as homework depending on the background and readiness of the students. In discussing the problems with graphs, try to emphasize to students the relationship between input and output as it is shown at the top of Activity 1.2.3b, rather than relying on the language of the “vertical line test” and “horizontal line test.”

**Exit Slip 1.2** can be completed after Activity 1.2.3.

**Activity 1.2.4a/1.2.4b Are Conics Functions?** involves another look at the conic sections introduced in Geometry. Consider showing the video <https://www.youtube.com/watch?v=GDHNoQHQmtQ> that demonstrates how to create each conic section (a circle, an ellipse, a parabola, and a hyperbola) by intersecting a plane and a pair of cones. (The video refers to the “angle of the generator”; you might need to explain what that means to students.) Another video that illustrates the effects of changing the parameters of ellipses using sliders, but does not require any downloads such as GeoGebra, is available at <http://www.mathopenref.com/coordgeneralellipse.html>.

Activity 1.2.4a uses GeoGebra, which is available as a free download from the website <https://www.geogebra.org/>. Students will need to access the software in order to complete this version of the Activity. If GeoGebra is not available to students, or if you prefer not to ask students to use GeoGebra, Activity 1.2.4b is a very similar version of this assignment that does not use the software. If you use Activity 1.2.4b, you might need to point out to students that they cannot make graphs of circles, ellipses, or parabolas and hyperbolas oriented horizontally using a graphing calculator because the graphing calculator does not allow them to enter an expression with a y2 term in it. After the assignment is complete, point out that any such expression cannot be entered because these expressions do not define the variable y as a function of x.

**Differentiated Instruction (Enrichment)**

Upon completing Activity 1.2.4a/1.2.4b, students will see that a circle in the form x2 + y2 = r2 does not define y as a function of x. Challenge your students to find a way to use a graphing calculator to graph a circle. (This can be done by graphing two functions, y = $\sqrt{r^{2}-x^{2}}$ and y = $-\sqrt{r^{2}-x^{2}}$.)

When Activity 1.2.4a/1.2.4b is complete, it is important to point out to students that these activities only consider one possible form of a hyperbola, the one with its major (or transverse) axis parallel to either the x-axis or the y-axis. This is the standard form of a hyperbola considered in Geometry. However, there are hyperbolas with a major (or transverse) axis that is not parallel to either the x-axis or the y-axis, and these can be functions. For example, the function f(x) = $\frac{1}{x}$ has the graph of a hyperbola.

**Differentiated Instruction (Enrichment)**

Rather than simply stating that it is possible for a hyperbola to be a function, challenge your students to find a function that has the graph of a hyperbola.

**Closure Notes**

Students should conclude from Activity 1.2.4a/1.2.4b that the circle and the ellipse cannot represent functions, and that the parabola and the hyperbola may or may not be functions, depending on their orientation. It is important to point out to students that sometimes in mathematics, we do want to consider the circle or the ellipse as a function (for example, in Calculus when we might consider the volume of a sphere or an ellipsoid). If we consider only half of a circle or an ellipse, we can consider the resulting graph to be a function.

Algebra 2 will focus on the study of various families of functions, and both Precalculus and Calculus look further at the behavior of functions. There are very few ideas in all of mathematics more powerful than “if I know this value (of the input), then I always can determine this value (of the output).” You might revisit the scenario from the beginning of Investigation 2 or describe some other scenario important to students (such as their monthly cell phone bill depending on the amount of data they use in a month) to emphasize the importance and pervasiveness of functions, both inside and outside of mathematics.

**Vocabulary**

Angle of a generator (for conic sections)

Area

Axis of symmetry

Circle

Circumference

Depreciation

Directrix

Domain

Ellipse

Foci of an ellipse, hyperbola, or parabola

Function

Function notation

Horizontal and vertical axes

Hyperbola

Major axis/minor axis of an ellipse

Natural domain

One-to-one (1-1) function

Ordered pair

Origin

Parabola

Parameter

Radius

Range

Relation

Standard form (of an equation for conic sections)

Vertex

**Resources and Materials**

**Activities 1.2.2, 1.2.3a, 1.2.3b, and either 1.2.4a or 1.2.4b should be completed in this Investigation by all students. Activity 1.2.1 is a review of material from Algebra 1; the material in this Activity can be reviewed and used as needed by the class.**

Activity 1.2.1 Function Review

Activity 1.2.2 What’s Reasonable?

Activity 1.2.3a Function Junction

Activity 1.2.3b Is The Function 1-1?

Activity 1.2.4a Are Conics Functions? (GeoGebra version)

Activity 1.2.4b Are Conics Functions? (non-GeoGebra version)

Graphing calculator/computer software with a graphing utility for all activities

Graph paper for all activities

Online access for Activity 1.2.1, 1.2.4a

Rulers as a straightedge

<https://www.geogebra.org/>

<https://www.youtube.com/watch?v=GDHNoQHQmtQ>

<http://www.mathopenref.com/coordgeneralellipse.html>.