**Unit 5: Investigation 7 (5 Days)**

**If unit 7 in Algebra one was not done (6 days)**

**Financial Mathematics**

**Common Core State Standards**

A-CED-1 Create equations and inequalities in one variable and use them to solve problems. *Include equation arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED-2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-LE-5 Interpret the parameters of an exponential function in terms of a context.

A-SSE-4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

**Overview**

 In this last investigation students will explore some personal financial mathematics with activities including fixed installment loans, such as car payments, mortgages, annuities. If students did not in Algebra 1 have time for investigation 7.5 then Activity 7.5.5 Compound Interest and Activity 7.5.6 on Doubling Times and Half-Life should be completed before proceeding. (You may have already completed as recommended Activity 7.5.5 before Investigation 2 in this unit.) In Activity 7.5.6 the work on investments and the car depreciation should at least be completed. If time permits the half-life activity could also be done.

In Investigation 2, students looked at compound interest problems with interest compounded annually, monthly, and daily. Compounding interest more frequently led them to the formula for compounding interest continuously, introducing the irrational (and transcendental) number e.

Students will again use the compound interest formula. Students will start with saving for a down payment on a car (short term savings) and see that making a regular deposit is similar to a compound interest situation. They will then look at a saving long term situation (for retirement) to see the power of compounding over time. They will return to a savings for a down payment on a house (long term planning). They will then examine a car payment situation, examine options and determine what they can afford, and lastly examine financing options. Students can be supplied the various formulas. The essence is that they can distinguish between the financial situations and determine which of various formulas is needed for a given application.

**Evidence of Success: What Will Students Be Able to Do?**

* Interpret and solve application problems that can be modeled by various financial formulas.
* Verify that a solution makes sense in the context of the problem.
* Be able to explain the power of exponential growth that comes with time.
* Translate a verbal description of a relationship into an equation or an inequality.

**Assessment Strategies: How Will They Show What They Know?**

**Exit slip 5.7.1** Students willapply the Future Value for an Annuity Formula to a short term savings problem.

**Exit slip 5.7.2** Students willapply the Future Value for an Annuity Formula to a long term savings problem.

**Journal Prompt 1** Why does the interest earned on an account with compounding always have to be at least as much as that earned from simple interest? If the money is invested only a short time, why can simple interest be used as an estimate of the compounded interest that would be earned?

**Journal Prompt 2** For a short term loan, why can we estimate the monthly payment by the amount of the loan divided by the term in months? For all loans why must the monthly payment be at least the amount of the loan divided by the term in months?

**Activity Sheet 5.7.1 Saving for a Down Payment (Part 1)** Students willapply the Future Value for an Annuity Formula.

**Activity Sheet 5.7.2 Think before You Buy that Drink** A group activity applies the Future Value for an Annuity Formula but also demonstrates the power of compounding over a long period.

**Activity Sheet 5.7.3 Saving for a Down Payment (Part 2)** Saving for a down payment of a house. This activity can also use the Future Value for an Annuity Formula.

**Activity Sheet 5.7.4 Mortgage Payments** will have students make an amortization table—a few rows.

**Activity Sheet 5.7.5 What Car Can I Afford?** will have students use the Monthly Payment of a Loan Formula to determine monthly car payments.

**Activity Sheet 5.7.6 Financing Options** will have students continue to use the Monthly Payment of a Loan Formula to determine monthly car payments in different scenarios as well as introducing the impact of a very good, good or poor credit score.

**Activity Sheet 5.7.7 Investment Opportunities—Expectations vs Reality** returns to compound interest and continuous compound interest applications and also has credit card problem which opens the door time permitting to address credit card interest and minimum payments and uses another mortgage formula. It can be used for review or exercises can be selectively assigned based on class needs.

**Launch Notes**:

Pose the scenario: Mary is shopping for a new car, and must decide which sales incentive to use when purchasing the car. She can choose 0.0% financing for up to 36 months but after 36 months she will owe 4.5% on any remaining balance, 0.9% financing for up to 60 months, or $2000 cash back with a 3.5% APR for 48 months. Mary can only afford a $450 monthly car payment. If the price of the car is $25,500, which is the best deal for Mary?

Next, suppose Mary has $3000 in the bank that she can put down as a deposit. Now, which is the best deal for Mary?

By the end of the investigation students should be able to return to this problem and solve it.

**Teaching Strategies**

**Activity sheet 5.7.1** focuses on using the Future Value of an Annuity Formula. It starts with looking at the finite compound interest formula. You might ask students what that formula is and what the variables represent before distributing the activity sheet. Suppose we make regular deposits (C) into a savings account. To find how much we will have in the future (F), use

Students will first use this formula as stated for Aaron, who has 6 months to save for his car and he knows he can save $100 at the end of each month. He desires to know how much money he will have, F when he goes to purchase his car. Then Hilary wants to take a cruise in two years and it will cost her $3000. She needs to find C. We can solve the formula for C to generate a new formula. It is listed in the resources below. But we also can just use the one above, replace concretely all values except C, and use our equation solving techniques to solve for C. Students will solve for C in Activity 5.7.5. Students needing challenge can do so now.

**Differentiated Instruction (Enrichment).** Have students solve for C for those situations where they must find a lot of values of C so it would make sense to change the subject of the formula. Students should solve for the variable C and get:

C=

The last scenario in Activity 5.1.1 is Finbar’s. He wants to save up to move to an apartment and will need a security deposit. He knows the amount he needs in the future and he knows how much he can save each moth so he wants to know how long it will take him to save the $900. Again students can change the subject of the formula but since we only have the one problem perhaps again solving concretely is the efficient choice. The abbreviation APR is used. Annual percentage rate is the rate without taking into account compounding within the year. You may want to contrast with APY, annual percentage yield or wait till later in this investigation to discuss APY.

**Differentiated Instruction (Enrichment).** Student can research the definitions of APR and APY. Students should be able to explain why as a borrower you might want the APY and not the APR quoted by banks but banks may want to use the APR so their loan rate looks lower. But now APR for loans should include fees too so as a borrower you may want both the APR and APY as you shop around. As a depositor you want to get the most interest so bankers will give the APY because it will include the effects of compounding.

**Activity 5.1.2** is intended to reinforce that the power of compounding lies with a long time period. They saw this in Algebra 1 and again in Unit 3 when exponential growth was compared with polynomial growth. In this activity they will see the power of compounding in long term savings. This activity, since it uses the formula from Activity 5.1.1, can be used as a group activity. Before any student does part 2 you may want to have the students decide which young lady will have more money when they turn 50, Jenell or Karina. It would be fun to have the students place their names on 3 by 5 cards and their choice of Janell or Karina and WHY. Then when the activity is completed see how many thought the $ 10 deposit would over power the $15 deposit and by how much. **Exit slip 5.7.1** can be used any time after Activity 5.7.1 has been discussed in class.

**Group Activity Sheet 5.7.2**. Students might do Part 1 in large class mode, then have students read the part 2 scenario and on a 3 by 5 card write down their name and the name of the young lady they think will have more money saved for the trip and perhaps even a guess as to how much money they think she will have saved, and why they chose the girl they did. Then in pairs or groups of three let them do all the computations.

**Journal Prompt 1** Why does the interest earned on an account with compounding always have to be at least as much as that earned from simple interest? If the money is invested only a short time, why can simple interest be used as an estimate of the compounded interest that would be earned? Students might respond, that since in compound interest situations you earn some interest earlier and the interest earned now becomes part of the principal and therefor the interest earns interest so the next calculation period has to generate a bit more interest. The one time both will be the same is when the compounding is once a year. For that situation you will only see more interest earned if the money is left for a second year. If money is left for only a short time, there is not much time for the interest to earn interest so using the simple interest amount provide a good estimate. If anything it will be a slight underestimate.

**Activity Sheet 5.7.3** applies the Future Value for an Annity Formula again to save for a down payment of a house. For Stem-intending students you might want to use the form of the formula that has C as its subject. Because this is still an application using the same formula as the earlier ones and not all students this age may be interested in a mortgage, this activity in the interest of time can be omitted or it can be used for homework. All students will see a mortgage application in the next activity and will examine an amortization table. **Exit Slip 5.7.2** can be used any time after activity 5.7.3 has been discussed.

**Activity Sheet 5.7.4** will introduce students to how an amortization formula is made. Intentionally, it does not need the Monthly Repayment of a Loan Formula. The focus is on if one has the monthly amount how does one determine the interest owed in a given month and of course then how much is left to be applied to the principal. The next activity will introduce the Monthly Payment of a Loan Formula. The focus is not on how the bank calculated that monthly payment, but to understand the concept of this repayment process.

**Differentiated Instruction (For Learners Needing More Help). In Activity 5.7.5** you may want to provide scaffolding to assist in the development of the change of subject from F to C in the Future Value of an Annuity Formula. You could have side by sides. On the left a concrete numerical problem with only C as a variable and then on the right side the steps for the formula.

**Activity 5.7.5** has student solve the Future Value of an Annuity Formula for C and then has them compare it to the Monthly Payment of a Loan Formula

M = amount of monthly payment

r = annual interest rate (written as a decimal)

n = number of compounding periods per year

t = number of years

P = principal on the loan

Students then apply the formula in varying car loan scenarios.

**Activity 5.7.6** has additional problems and also has students examine the impact poor credit may have on a loan payment. If time permits the formula for credit card payoff has been included in the resources and it can be discussed and applied or used as an enrichment activity.

**Differentiated Instruction (Enrichment).** Have students make up an application problem that needs one of the formulas discussed in this investigation. They can then be solved for homework or as a review for the final exam.

**Differentiated Instruction (Enrichment).** Students can use the formula for a credit card payoff in a problem setting.

**Journal Prompt 2** For a short term loan, why can we estimate the monthly payment by the amount of the loan divided by the term in months? For all loans why must the monthly payment be at least the amount of the loan divided by the term in months? Students might say, for a short term note the effect of compounding will not be severe so for a quick albeit under estimate the amount of the loan divided by the number of months will give a quick yet fairly good underestimate. For all loans a person has to repay the principle borrowed so even with no interest charged you would owe the amount of the loan divided by the number of months for the loan.

**Activity Sheet 5.7.7 Investment Opportunities—Expectations vs Reality** returns to compound interest and continuous compound interest applications and also has credit card problem which opens the door time permitting to address credit card interest and minimum payments and uses another mortgage formula. It contains a piecewise function, and students need to solve the initial amount as well as find an interest rate. It can be used for review or exercises can be selectively assigned based on class needs.

**Closure Notes**

On the final day of this investigation have student break into predetermined groups and have each group work on a part of the opening day problem. One can compare Mary in the 36 months with 0% and the other 48 months with .9%, and another group can work on the .9% and the 3.5% with cash back. Other groups can break up in the same two ways except they will have the down payment. Groups can then report out and the best deal can be selected for Mary. Students at the end should be able to explain how the two formulas used in the investigation can apply to so many different situations.

**Vocabulary**

Annual Percentage Rate (APR)

Annual Percentage Yield (APY)

Annuity

Credit card

Car loans

Compound interest

Continuous compound interest

Future value (FV),

Interest rate

Monthly payment

Mortgages

Number of payments to payoff credit card debt (N),

Present value (PV),

Principal

Regular payment amount (PMT),

Simple interest

**Resources and Materials**

**All activities should be completed except 5.7.3 and 5.7.7 can be selectively assigned**

Activity Sheet 5.7.1 Saving for a Down Payment (Part 1)

Activity Sheet 5.7.2 Think before You Buy that Drink

Activity Sheet 5.7.3 Saving for a Down Payment (Part 2)

Activity Sheet 5.7.4 Mortgage Payments

Activity Sheet 5.7.5 What Car Can I Afford?

Activity Sheet 5.7.6 Financing Options

Activity Sheet 5.7.7 Investment Opportunities—Expectations vs Reality

Graphers

Formula used to determine the money regular payments (C) into a savings account when you know the future value (F).

Annual percentage yield (APY) is the effective annual interest rate earned in a given year.

Present value (PV) is the amount of principal needed to reach a certain future value (A).

Present value (PV) is the amount of principal needed to reach a certain future value (A).

Do you have credit card debt? This formula shows just how long it can take to pay off a credit card debt, which illustrates why making just the minimum payment is not effective. To find out how many, N, (fixed) payments PMT required to pay off a debt such as a credit card, is given by

How do car dealers figure out payments on a car loan? If monthly payments are made, *n=12.*

This same formula may be applied to mortgage payments for a house or boat.