**Unit 5: Investigation 5 ( 3 Days)**

**Curve fitting with Exponential and Logarithmic Functions**

**Common Core State Standards**

A-CED-1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED-2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-BF-3 Identify the effect on the graph of replacing by and for specific values of (both positive and negative); find the values of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-LE-5 Interpret the parameters of an exponential function in terms of a context.

**Overview**

Students will develop a deeper understanding of regression equations and predictions in this investigation. They will build on their understanding from Unit 5 in Algebra 1. They will do some work by hand as well as use the technology of your choice. This investigation also provides an opportunity to practice algebra skills under the guise of obtaining equations that model data. In this investigation students will determine which curve, or model, best fits a set of data points. From a scatterplot, the data may appear to be linear, exponential, logarithmic or other. Students will extend beyond just being able to model with lines effectively. Since two points determine an exponential of form f(x) = abx (as they verified in Algebra 1) as well as a linear function, students will first explore passing an exponential though two points. In unit 7 of Algebra 1 they used trial and error to fit an exponential and used their knowledge of the fact that the parameter *a* is the y-intercept and then tried varying values of *b* to get a relatively good fit. The first activity will mirror the work done in Unit 5 of Algebra 1 with lines. They will select two points, not necessarily where one is the y-intercept, that they feel will permit them to pass the best exponential curve to model the data. They will get an algebra workout in doing so. Later in the investigation they will use technology and use the exponential regression provided by technology and they will also be introduced to the logarithmic regression feature on their technology for data sets best modeled by a member of the logarithmic family. Lastly an activity that can be used, time permitting, will have them graph log y vs x or ln y vs x and if the scatter plot looks linear, transform the equation and obtain an exponential equation from the linear one in the variables log y and x. This activity provides another opportunity for algebra manipulation and to be able to explain why an r value (correlation coefficient) appears on their technology screen with the exponential regression equation.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Given a set of data points, determine whether the model should be an exponential, linear, logarithmic function or neither or perhaps piecewise.
* Determine the exponential or linear equation through two points without technology.
* Find an exponential, linear or logarithmic equation that best fits a data set using technology.
* Interpret the parameters of the model and make predictions.
* Evaluate a model and adjust the domain appropriately.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 5.5** has one problem where students must determine the best fit for the data points.
* **Journal Prompt 1** You have found an exponential equation through two points as well as used technology to find an exponential regression. List some advantages and disadvantages of each method to model a set of data that is exponential.
* **Activity 5.1.1 United States Census** provides U.S. Census data and asks students to create, use and find a mathematical model for data that is exponential for a while. Students will be solving a nonlinear system. The activity also demonstrates the need to consider piecewise functions again.
* **Activity 5.5.2 Modeling the Population of the United States** continues to work with the U.S. census data but develops a piecewise defined function where both pieces are exponential. Students are provided with additional opportunities to solve a nonlinear system.
* **Activity 5.5.3 Superbugs and The Spread of Diseases** provides other contexts to evaluate a function that uses the base e in its definition, to solve a nonlinear system and to use the natural logarithmic function when given an output and the problem requires one find the input.
* **Activity 5.5.4 The Cost of a Used Car** has students work with data that is best modeled by a logarithmic family member. Students continue to solve nonlinear systems. They will also use technology to obtain a logarithmic regression.
* **Activity 5.5.5 Which Model Should I Use?** uses a data set that students model, using technology, with a linear equation, exponential, logarithmic and power and discuss the pros and cons of each model. They will realize modeling is not an exact science and also not just a matter of pushing a few keys on a grapher or equivalent technology.
* **Activity 5.5.6 Fitting an Exponential Function by Linearizing the Data** (and optional: Fitting a Power Function by Linearizing the Data) will provide students with an algebra work out and answer the question where does the r value come from since I am dealing with a curve. The students will also see how technology gives them an exponential or power regression. They will get continued practice in solving a nonlinear system of equations.

**Launch Notes**

Refer to the puppy data from Algebra 1 Activity 7.4.3 from ctcorestandards.org Algebra 1

|  |  |
| --- | --- |
| **Age (days)** | **Weight (kg)** |
| 0 | 1.48 |
| 10 | 1.93 |
| 20 | 2.5 |
| 30 | 3.18 |
| 40 | 4.09 |
| 50 | 5.23 |
| 60 | 6.82 |
| 70 | 8.64 |

Just give students the first and third rows of data. Remind students that weight is a function of age and you are assuming that the function is exponential because in Algebra 1 they were able to obtain a reasonable fit by trial and error. Using the given data, have them write an equation of the form

reminding them if necessary that they know the value of *a* already and to solve the equation to find the value of . Using the function they created, have them predict the weight of the puppy when he is 30 days old. Then 50 days old. From Algebra 1 they should remember that the model does not work forever. Ask them why.

Now have them repeat the process but use the rows for 10 days and 30 days. Using the given data, they need to write two equations of the form

and solve them to find values for and . Go through this in large class discussion and again make some predictions.

**Teaching Strategies**

**Activity 5.5.1** continues the Launch Activity by giving them a new table of values. For additional practice they solve for both *a* and *k* even though they know the value of *a*. This activity should make a good group or pair activity.

**Group Activity 5.5.1** provides US Census data and asks them to create, use and find a mathematical model for data that is exponential for a while. Students will be solving a nonlinear system.

**Differentiated Instruction (For Learners Needing More Help)**

Using the scatterplot of the original data, see that by simply looking at the data we cannot determine if its growth is exponential. Look at examples using technology. For example, look at y = e^x and y = x^3 + 1. For small values of x the graphs of the two functions are very similar.

**Differentiated Instruction (Enrichment)**

Using the scatterplot of the data, from 1790 through 1990, students could define a piecewise function and provide the improved model in response to part 5 number 3 of Activity 5.5.1. Activity 5.5.2 will do this but it is a directed activity with scaffolding. Or you may care to have students work on activity 5.5.2 but use 3 pieces instead of the two used in the activity and then discuss which model they feel is best.

**Activity 5.5.2** continues to work with the U.S. census data but develops a piecewise defined function where both pieces are exponential. But since students are only using two points some point choices are better than others which reinforces the work they did in Algebra 1 with linear modeling and the launch where students obtained different equations when they chose different point pairs. Again the stress is on: create a model, test it, if not good, retry and when the fit is acceptable make predictions. Two additional data set from Algebra 1 unit 7 are repeated in the teacher resources and can be used for additional practice as needed for finding an exponential through two points or later after Activity 5.5.5 to find the regression equation with technology.

**Activity 5.5.3 Superbugs and The Spread of Diseases** provides other contexts to evaluate a function that uses the base e in its definition, to solve a nonlinear system and to use the natural logarithmic function when given an output and the problem requires one find the input. The first 6 exercises can be assigned for homework. With many classes you may want to do number 7 and 8 in class.

**Activity 5.5.4** has students work with data that is best modeled by a logarithmic family member. The modeling form is given to students and the two-fold objective is to have students recognize that the logarithmic function can be used to model data and what they need to look for in a scatter plot and to continue to work with systems of nonlinear equations.

**Differentiated Instruction (Enrichment)** Students can be provided with data that is best modeled by a power regression, and graph log *y* vs log *x* and see that it is linear. The data may be fit by a power function because taking the natural logarithm of both sides yields .

Let and the equation is of the form Y = b + mX which shows that when plotting (ln x, ln y), power data will appear

linear. Now the challenge is to undo the transformed data since students will have an equation for the linear graph ln *y* = ln *a* + mln *x*

*e*lny = *e* ln a + mlnx = *e* ln a *e*mlnx

*y* = *e* ln a (*e*ln x)m

*y* = a*x*m

In **Activity 5.5.5** students use the regression feature of their technology. Students used the linear regression feature in Algebra 1 so they should remember the procedure and the only difference will be selecting the power, exponential or logarithmic choice. The teacher might want to have the students do an exponential regression for the puppy problem to refresh memories and key strokes before working on this activity. Two other data sets from Algebra 1 are provided in the resources. Either of these can be used for additional practice with technology and for obtaining and interpreting the parameters the regression equation.

**Differentiated Instruction (Enrichment)**

For activity 5.5.5, in addition to context and r – values students could research what a residual is and explore residuals and their contribution to selecting the best model for the given data set.

**Journal Prompt 1** You have found an exponential equation through two points as well as used technology to find an exponential regression. List some advantages and disadvantages of each method to model a set of data that is exponential. The students should mention that the regression equation from technology is unique so any predictions they make will result in the same answer. Passing a curve through two points will yield different equations. If you only have two points and especially one is the y-intercept determining the equation by hand might be faster.

In **Activity 5.5.6** students will linearize exponential data. In earlier activities they have already noticed that if the dependent variable has a logarithmic scale and that when the data is plotted, a linear pattern results. Students will transform data that is suspected exponential to see if there is a linear pattern. When there is, they will get a regression line and then undo the transformation to get the exponential equation for the original data. If time is short this activity can be omitted. It may be omitted for non-stem intending students as well.

**Differentiated Instruction (Enrichment)**

In activity 5.5.6, there is an optional section II that has students explore data they know already was modeled with a power function earlier in the unit. They linearize the data and obtain a linear equation with variables log *x* and log *y* and then transform the linear equation. They then get a power regression from the TI and see that the power regression matches the equation they obtained by linearization of the data.

**Differentiated Instruction (Enrichment)** Students can also explore the logarithmic regression where we graph *y* vs ln *x* to linearize the data and find the best linear fit, *y* = a ln*x* + b for the transformed data.

**Closure Notes**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| y | 1.6 | 2.7 | 4.4 | 6.4 | 8.9 | 13.1 | 19.3 | 28.2 | 38.2 | 48.7 |

The table above lists the number y (in millions) of cell-phone subscribers in the US from 1988 to 1997, and x is the number of years since 1987. Have half the class find an exponential fit by plotting the data and selecting two points and find the exponential function through those two points and the other half of the class enter the data in their graphers and find a curve of best fit. In terms of this problem have students discuss the advantages and disadvantages of each method.

**Vocabulary**

Best Fit

Curve Fitting

Exponential Function

Good Fit

Logarithmic Function

Modeling

Power Function

Scatterplot

**Resources and Materials**

**All activities should be completed but section II of Activity 5.5.6 is optional**

Activity 5.1.1 United States Census

Activity 5.5.2 Modeling the Population of the United States

Activity 5.5.3 Superbugs and the Spread of Diseases

Activity 5.5.4 The Cost of a Used Car

Activity 5.5.5 Which Model Should I Use?

Activity 5.5.6 Fitting an Exponential Function by Linearizing the Data

Graph paper for scatterplots

Technology (Calculators)

Data from unit 7.3.4

In 1971 Starbucks opened in Seattle. In 1987 it began to expand its locations. (If you want more information you can visit www.starbucks.com/aboutus/timeline.asp) The table of data below provides the numbers of stores in business for the years 1987 – 2004.

|  |  |  |
| --- | --- | --- |
| **Year** | **Years since 1987** | **Number of stores** |
| 1987 | 0 | 17 |
| 1988 | 1 | 33 |
| 1989 | 2 | 55 |
| 1990 | 3 | 84 |
| 1991 | 4 | 116 |
| 1992 | 5 | 165 |
| 1993 | 6 | 272 |
| 1994 | 7 | 425 |
| 1995 | 8 | 676 |
| 1996 | 9 | 1015 |
| 1997 | 10 | 1412 |
| 1998 | 11 | 1886 |
| 1999 | 12 | 2135 |
| 2000 | 13 | 3501 |
| 2001 | 14 | 4709 |
| 2002 | 15 | 5886 |
| 2003 | 16 | 7225 |
| 2004 | 17 | 8337 |

In this table the first column gives us the year and the second column tells us how many years have passed since 1987.

Data from unit 7.3.4

During the 20th century much progress was made toward detecting and treating various cancers. The table below shows the rate of deaths from stomach cancers among women in the United States from 1930 through 1990. (Source: <http://www19.homepage.villanova.edu/alice.deanin/courses/Mat7310/exponential.htm>)

|  |  |  |
| --- | --- | --- |
| **Year** | **Years since 1930** | **Number of deaths per 100,000 women** |
| 1930 | 0 | 28 |
| 1940 | 10 | 21 |
| 1950 | 20 | 13 |
| 1960 | 30 | 9 |
| 1970 | 40 | 6 |
| 1980 | 50 | 5 |
| 1990 | 60 | 4 |

Enter the data into your calculator and choose an appropriate window to view the data. The data should lie along a downward sloping curve indicating that an exponential function may be the right choice for a model.