**Unit 2 Congruence, Constructions, and Proofs**

**(3-4 Weeks)**

**UNIT PLAN**

In keeping with the Common Core Standards, the fundamental idea of congruence is defined in terms of transformations. The properties of transformations discovered in Unit 1 become postulates as we begin to construct a formal system of geometry.

In **Investigation 1** the definition of congruence is used to establish properties of congruent segments, angles, circles and polygons. The symbol ($≅)$ is introduced and its use is illustrated.

In **Investigation 2** the first theorems about congruent triangles, SAS and ASA are proved using transformational methods.

Isosceles triangles are introduced in **Investigation 3**. We prove that if two sides of a triangle are congruent, then the angles opposite these sides are congruent, and conversely. Isosceles triangles are used in the proof of the **SSS Congruence Theorem** in Investigation 4.

Relationships among angles formed by intersecting lines and two parallel lines cut by a transversal are discussed in **Investigation 5**.

We have now proved a small set of theorems that can be used to justify many of the traditional compass and straightedge constructions of Euclidean geometry. **Investigation 6** is open ended. Students are challenged to figure out how to perform certain constructions, for example to construct the perpendicular bisector of a line segment. In **Investigation 7** they use the theorems previously proved to justify their own constructions or one that is given to them.

Students then apply what they have learned in the **Performance Task** to create their own design.

**Essential Questions**

* What are the properties of congruent figures?
* How can two triangles be proved congruent?
* What relationships hold among angles formed by parallel lines and a transversal?
* How can your construct geometric figures with a compass and straightedge?
* How can you prove the constructions work?

**Enduring Understandings**

Theorems about congruent triangles may be proved using transformations and applied to justify constructions.

**Unit Contents**

Investigation 1 Identifying Congruent Figures (2 days)

Investigation 2 SAS and ASA Congruence (2 days)

Investigation 3 Isosceles Triangles (2 days)

Investigation 4 SSS Congruence (2 days)

Investigation 5 Vertical Angles and Parallel Lines (2 days)

Investigation 6 Constructions (3 days)

Investigation 7 Proving That Constructions Work (2 days)

Performance Task: Create a Design (1 days)

End-of-Unit Test (2 days including review)

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. **Construct viable arguments and critique the reasoning of others.**

4. Model with mathematics.

5. **Use appropriate tools strategically.**

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Standards Overview**

* Understand congruence in terms of rigid motions
* Prove geometric theorems
* Make geometric constructions

**Standards**

G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

G-CO.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.*

G-CO.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

**Vocabulary**

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| acute triangle |

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| alternate exterior angles |
| alternate interior angles |
| altitude (of triangle) |
| base (of isosceles triangle) |
| base angle (of isosceles triangle) |
| compass |
| conclusion |
| congruentcongruence symbol ($≅$) |
| consecutive (sides, angles) |
| construction |
| convex (polygon) |
| corresponding angles (formed by tranversal) |
| corresponding parts |
| dart |
| drawing |
| equilateral triangle |
| Euclidean construction |
| hypothesis |
| included angle |
| included side |
| interior angle of a polygon |
| isosceles triangle |
| kite |
| leg (of isosceles triangle) |
| linear pair of angles |
| median (of triangle) |
| non-convex (polygon) |
| obtuse triangle |
| parallel lines |
| pependicular bisector |
| polygon |
| proof |
| proposition (from Euclid’s *Elements*) |
| right triangle |
| same-side exterior angles |
| same-side interior angles |
| scalene triangle |
| side of a polygon |
| straightedge |
| supplementary angles |
| transversal |
| vertex angle (of isosceles triangle) |
| vertical angles |

**Postulates**

**Point-Line Postulate.** Between two points it is possible to construct exactly one line. Two lines intersect in at most one point.

**Plane Separation Postulate:** If a line *l* lies in a plane. then every point in the plane not on *l* lies on one side or the other side of *l*. If *B* lies on the opposite side of *l* from *A,* then segment $\overbar{AB}$ intersects line *l* in one point. (Also mentioned in Unit 1)

**Linear Pair Postulate:** If two angles form a linear pair, then they are supplementary

**Parallel Postulate:** Through a point not on a line there is exactly one line that can be drawn parallel to the given line.

**Theorems**

**SAS Congruence Theorem**: If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent to each other.

**ASA Congruence Theorem:** If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent to each other.

**Isosceles Triangle Theorem:** If two sides of a triangle are congruent, then the angles opposite these sides are congruent.

**Isosceles Triangle Converse:** If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

**Equilateral Triangle Theorem:** If all three sides of a triangle are congruent, then all three angles are congruent.

**Equilateral Triangle Converse:** If all three angles of a triangle are congruent, then all three sides are congruent.

**SSS Congruence Theorem** If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

**Vertical Angles Theorem:** If two lines intersect, pairs of vertical angles are congruent.

**Parallel Lines Corresponding Angles Theorem:** If two parallel lines are cut by a transversal, then pairs of corresponding angles are congruent.

**Parallel Lines Alternate Interior Angles Theorem:** If two parallel lines are cut by a transversal, then pairs of alternate interior angles are congruent.

**Parallel Lines Alternate Exterior Angles Theorem:** If two parallel lines are cut by a transversal, then pairs of alternate exterior angles are congruent.

**Parallel Lines Same Side Interior Angles Theorem:** If two parallel lines are cut by a transversal, then pairs of same-side interior angles are supplementary.

**Parallel Lines Same Side Exterior Angles Theorem:** If two parallel lines are cut by a transversal, then pairs of same-side exterior angles are supplementary.

**Constructions (with Proofs)**

**Equilateral Triangle:** Given a segment, to construct an equilateral triangle with the segment as a side

**Bisect an Angle:** Given an angle, to construct the ray that bisects the angle

**Perpendicular to a Line at a Point on the Line:** Given a line and a point on the line to construct another line through the given point that is perpendicular to the given line

**Perpendicular to a Line from a Point not on the Line:** Given a line and a point not on the line to construct another line through the given point that is perpendicular to the given line

**Perpendicular Bisector of a Segment:** Given a segment to construct its perpendicular bisector

**Triangle Given Three Sides:** Given the lengths of the three sides of a triangle, to construct the triangle.

**Duplicate an Angle:** Given an angle to construct another angle congruent to it.

**Assessment Strategies**

**Performance Task**

Students will use compass and straightedge to create a design and list the steps used in the construction.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* End of Unit test