**Final Exam Review Bank**

Teacher Note: Some problems are listed solely under one unit but will use concepts from earlier units and others are placed in a combination of units emphasizing a concept across units so please read through the entire selection. A formula sheet is available for students to use.

Units 7 an d 8 have the problems identified by activity number because we know the entire units may not have been completed.

**Unit 1**

1. On the set of axes provided below, graph the solution set of 2*x* – 4*y* < 10.





1. Give examples of each of the following:
2. A relation that is not a function defined by a table of values.

Answers will vary but need a table with two identical input values but different outputs.

1. A relation that is not a function defined by a graph.

Any graph that fails the vertical line test.

1. A relation that is not a function defined by a symbolic statement (an equation).

Any equation such as x = y2 or the equation of any circle and students met the unit circle x2 + y2 = 1 in unit 6.

1. Now return to parts a – c and explain why the relations you have defined are not functions.

Each table should have x1 = x2 but y1 ≠ y2. Graphs should fail the vertical line test, and students should state that the equations permit you to have at least one pair of ordered pairs where x1 = x2 but y1 ≠ y2.

1. Find the inverse of the following functions:

B.

1. A. The function does not have an inverse. Find a suitable restriction of the domain of the function so that it does have an inverse.

D = [2, or some might select (-,2]  
  
B. Find the inverse of with the restriction you gave in question A.

or if they select the restriction (-,2]

1. Knowing that a function is odd can make it easier to graph the function. Why?

You can reflect each point in the first quadrant about the origin (x,y) into (-x,-y) or each in the second quadrant to the fourth quadrant (-x,y) into (x,-y) for x > 0 and y > 0.

1. Is (4,-5) a solution of the system 7x – 4y > 3 and 2x + 6y < 28. Explain.

Yes. 7(4) – 4(-5) = 48 > 3 and 2(4) + 6(-5) = -22 < 28.

**(Note: Questions 7, 8 are for students who have studied Investigation 5 in Unit 1. ☺)**

1. In question #3B above, you found the inverse of the function . Verify that this function and the inverse function you found in question #3B are inverses by finding the composition of the functions.

(x)) = f(2(

1. You are given the functions and
2. Find the function f ◦ g. f ◦ g =
3. Find the function g ◦ f. g ◦ f =
4. The temperature at which water boils decreases as altitude increases, a fact you must take into consideration if you are cooking at high altitudes. Most cookbooks consider 3000 feet above sea level to be high altitude but even at 2000 feet above sea level the boiling point of water is 208 degrees F. At higher altitudes, cooking times must be increased to compensate for the lower boiling point. Check the back of a cake mix for high altitude cooking instructions. With each 500-foot increase in elevation, the boiling point of water is lowered just under 1ᵒF.
5. Model this information with an equation.
6. What variable should be the independent variable? Height above sea level
7. Which variable should be the dependent variable? Boiling point of water
8. When you are 7500 feet above sea level, at what temperature will water boil? 198ᵒ
9. Suppose is the boiling point of water in ᵒF at an altitude of *h* feet above sea level during standard atmospheric conditions. What is the meaning in the context of this situation of ?

When the boiling point of water is 198 degrees F you are at an altitude of 7500 feet.

1. What is the meaning of ?

The boiling point of water at sea level is 212 degrees F

1. gives the value in dollars of *q* quarters.
2. Find and describe what it means in this problem setting.

$6.75 is the dollar value of 27 quarters.

1. If Jon has a pile of quarters worth $47.75, how many quarters are in his pile?

191 quarters

**Units 1 – 6**

1. Given the functions and, find:
2. (f + g) (2) 5/7
3. (f – g) (-1) 33/8
4. Domain of f + g is (-
5. Domain of f – g is (-
6. Given the functions and over [0, 2π], find:
7. (fg) (π) -4π-3
8. (0) 1/43
9. Domain of fg is [0, 2π]
10. Domain of f/g is [0, π/2)U (π/2, 3π/2)U (3π/2, 2π]
11. Which of the following functions are one-to-one? Provide some evidence your answers?

Functions a, c, e, g, and h are one to one. Answers will vary. Students could graph and claim the graphs satisfy the horizontal line test. They could argue that the logarithmic and exponential families are inverses of each other so must be one to one. They could argue that f, b and d have symmetry with respect to the y-axis so could not be one-to-one. They could use concrete point pairs to by counterexample prove which functions are not one-to-one.

**Unit 2**

1. The height in feet of a baseball after *t* seconds is given by
2. When will the ball hit the ground? Give exact answer and approximate if needed to tenth of a second. (10 + √103) /4 Or about 5.0 seconds
3. When will it be 67 feet above the ground? 1 and 4 seconds
4. Find the maximum height and how long it takes to reach the max height. The ball

will reach a maximum height of 103 feet in 2.5 seconds.

1. Solve over the real numbers:

1. x2 – 9x + 8 = 0 {8, 1}
2. 4x2 + 9x + 8 = 0 No reals
3. Solve over the complex numbers:
4. 36x2 – 25 = 0 {5/6,-5/6}
5. 36x2 + 25 = 0 {-5i/6, 5i/6}
6. a. Is the operation of addition for the reals commutative? Yes
7. Provide one concrete example. 7 + 4 = 4 + 7. Both are 11.
8. Is the operation of subtraction for the reals commutative? Justify your answer.

No. Answers may vary. For example, 5 – 3 ≠ 3 – 5

1. A rancher has 1200 feet of fencing available to enclose a rectangular pasture for his cattle. One side of the pasture will be along a river and he will not place fencing on that side of the pasture. He wants his cattle to have the maximum area they can to graze.
2. Express the area as a function of *w*, the width of the rectangle. The length runs parallel to the river and the two remaining sides have width *w*.

1. Find the width of the rectangle that will give the cattle the maximum grazing area.

The dimensions that maximize the area are 300 feet by 600 feet.

1. Convert the function to standard form.

1. Convert the function to vertex form.

2 – 26

1. Find the discriminant of the function and describe what the discriminant says about zeros of the function and the x-intercepts.

Discriminant = -64. There are no (real) x-intercepts but there are 2 complex zeros -1 + 2i and -1 – 2i . Teacher note: Students do not have to list the complex zeros.

1. The you are driving a car on dry pavement, the distance it takes to stop the car depends on the speed you are traveling, the amount of time it takes you to react (from the time you decide to stop until your foot fully applies the brakes), and the friction between the tires and the road. A common function to describe the stopping distance is

where *v* is your velocity (in miles per hour) and is the total distance it takes to stop the car.

1. Give a reasonable domain and range for this function. [Hint: What is the fastest speed a car might travel?]

Answers will vary; a typical passenger car has a top speed of maybe 140 miles per hour, which would give the domain as [0, 140] and the range as [0, 660] approximately.

1. Determine the distance it takes for a car to stop if it is traveling at 60 miles per hour.

) = 172.44 so about 172 feet

1. A police investigator measures the skid marks for a car as being the full length of a football field, which is 300 feet. At what speed was the car traveling?

1. Does the function have an inverse? If you think does have an inverse, describe the meaning of . If you think explain why it does not.

does not have an inverse because it is not a 1-1 function.

**Unit 3**

1. Beth wants to make a box out of a piece of cardboard she found. The cardboard is 8 inches by 10 inches. She does not need a top. If she lets x represent the side of each square cut out. Find:
2. An expression in factored form that represents the volume of the box, V(x).
3. The domain of the function in this applied problem. (0,4)
4. The x-intercepts of the function without considering the applied problem restrictions.

0, 4, 5

1. Sketch the graph on your grapher and list the window you are using.

Answers will vary but the y max of the window should be at least 53.

1. Use your graph to estimate the maximum possible volume of the box and the dimensions of the square cut out. Round to the nearest tenth. .
2. If 4 + 3i is a zero of a cubic polynomial, what other number must be a zero of the cubic?

4 – 3 i\_\_

1. The graph of the function, , is given below. Using the graph, answer the following questions.

|  |  |
| --- | --- |
| a. What conclusion can you make about the sign of *a*? Explain your answer.  *a* must be positive since *f(x)–>+∞* as *x –>+∞.*  b. Find the exact values of *a*, *b*, *c*, and *d* if the point (7,-4) is a point on the graph.  *f(x)=a(x–5)2(x–8) = a(x3–18x2+105x–200)*  Using the point *(7,–4)* we get  *-4=a(7–5)2(7–8) = 4a.* Therefore*, a=1,*  and *f(x) = x3–18x2+105x–200* |  |

1. A quartic polynomial function, *f(x),* has two x-intercepts and a positive leading coefficient. Must *f(x)* have an absolute minimum? If yes, explain why and determine as much as you can about the sign of that minimum?

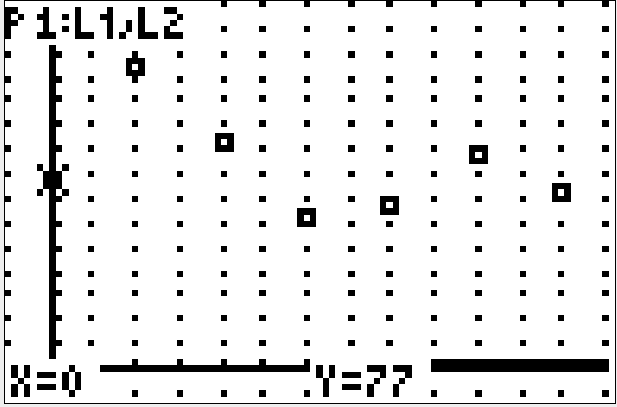
If a quartic function has two x-intercepts and a positive leading coefficient, the end behavior of the function indicates that *f(x) –>+∞ as x–>+/–∞.* Since *f(x)* as two x-intercepts, it must be true that either the graph of *f(x)* crosses the x-axis at each intercept or is tangent to the x-axis at each x-intercept. Therefore, there must be an absolute minimum which is less than or equal to 0.

1. The table below shows the average number of vehicles per minute that on a typical week day go through the EZ Pass lane on the Tappan Zee Bridge in New York during the 12-hour period from 6:00 am to 6:00 pm.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Hour | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| Cars per minute | 77 | 123 | 92 | 62 | 68 | 88 | 73 |

* 1. Explain how you decided what type of polynomial function you would choose to model the number of cars per minute that were going through the EZ Pass lane as a function of the hour?
  2. If we let t = 0 be 6:00 am, use your calculator to create a polynomial function, *f(t),* to model the average number of vehicles during that time.
  3. Use the model to predict the number of cars per minute going through the bridge’s EZ Pass lane at 1:00 pm.
  4. Would your model be effective in predicting the number of cars per minute passing through the EZ Pass lane at 12:00 midnight? Explain your answer.

The scatter plot for the data is*:*

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*a.* Using the data from the scatter plot, we see that there are 3 turns in the data, thus a quartic equation is the best function to model the data.

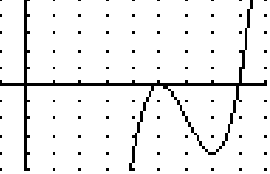
b. Using quartic regression we obtain a quartic polynomial function approximately equal to*: f(t) = -.117x4+3x3–24.2x2+60x+77.1*

c. At 1:00 pm, t = 7. Therefore, f(7) equals approximately 59 or 60 cars per minute passing through the EZ Pass lane depending on rounding of the coefficients.

d. The model would not be useful in predicting the number of cars going through the EZ Pass lane after midnight since t = 18 at that time and the model would give an negative number, which is not realistic.

1. The zeros of the polynomial function are the lengths of the sides of a triangle. Find the area of the triangle. (version 1)

Alternate number 28. The zeros of the polynomial function given by the graph below are the lengths of the sides of a triangle. Find the area of the triangle. (version 2: scaffolded with x-intercepts providing the zeroes.)



Note: This question comes from the *Mathematics Teacher* Calendar Problem of September 3, 2014.

(version 1).The student would need to find the x-intercepts algebraically. For this cubic function there is a double root at 5 and a single root at 8. Therefore, the sides of the triangle are 5, 5, and 8, which form an isosceles triangle. Dropping the altitude from the vertex, you form two right triangles with hypotenuse 5 and side 4. Using Pythagorean Theorem, the third side is 3, which makes the area, A=1/2 •8•3 = 12 square units.

(version 2). The students see the x-intercepts and that there is a double root at 5 because the graph is tangent to the x-axis at that point. The other x-intercept is 8, so the sides of the triangle are 5, 5, and 8. Therefore proceed as above to find the area of the triangle.

**Unit 4**

1. A rectangle has an area of 8 square meters.
2. Express the perimeter of the rectangle as a function of the width *w*.

1. Find the domain of the function in this applied problem.

(0,

1. Sketch the graph on your grapher.
2. For which value of w is the perimeter a minimum? You may have to use your graph to approximate the answer.

w = 2.83m has a minimum perimeter of 11.31m

1. What is the length of the rectangle?

2.83m

1. A trapezoid has an altitude of and bases of measure and .
2. Find an expression for its area.

1. If the area is 2 square centimeters, what is the value of x and what are the measures of the bases and the altitude? Give both exact and rounded to the nearest hundredth values.

x is 2√3. The bases are and cm and the altitude is √3 cm. (Approximately 1.73, 2.15 and 0.15 cm)

1. A family is planning a “destination wedding” where the entire wedding party flies to Hawaii. They are able to charter a flight to Hawaii for $50,000. There are 20 people in the wedding party who will fly on the charter flight. Since the plane seats up to 80 people, the family decides to ask some of their friends if they want to fly on their charter. The function describes the cost per person on the flight as a function of p, the number of their friends who agree to fly on the charter.  
   a. What is the domain of C(p)?  
   [0, 60]  
     
   b. What is the range of C(p)?  
   [625, 2500]  
     
   c. If 30 friends fly with the wedding party, find the cost per person on the flight.  
   C(30) = 1000  
     
   d. Find an equation for C-1(p).  
      
     
   e. Find the value of C-1(1250), and interpret what this value means.  
   C-1(1250) = 20; if the cost per person is $1250, then 20 friends flew on the charter.
2. Solve over the set of real numbers.
3. {4, -3}
4. 47/7
5. Perform the indicated operations and list any needed restrictions on the variables.
   1. x ≠ 0, 3
   2. x ≠ -7
   3. x ≠ -1, 3
   4. x ≠ 0, 4
6. A container holds n balls numbered 1, 2, 3, …, n and only one ball has the winning number.
   1. Write an expression for drawing the winning ball. 1/n
   2. A second container holds 5 fewer balls than the first container but has 3 winning balls. Write an expression for drawing a winning ball from the second container.
   3. A third container holds 7 more balls than the first container and has 2 winning balls. Write an expression for drawing a winning ball from the third container.
   4. Write an expression for drawing a winning ball from the first container, and a winning ball from the second container, and a winning ball from the third container.

or

1. The weight of an object on Earth is directly proportional to the weight of the object on Mars. A person who weighs 120 pounds on earth will weigh 45.2 pounds on Mars.

* 1. How much would a 200 pound astronaut weigh on Mars? 75 and 1/3 pounds
  2. The weight of an object on Earth is also directly proportional to the weight of the object on Jupiter. The 120 pound person would weigh 286.6 pounds on Jupiter!!! How much will the 200 pound astronaut weigh on Jupiter? 477 and 2/3 pounds

1. Driving at 55 mph, it takes about 2.75 hours to get Boston from Waterbury Connecticut. Is the time it takes to drive to Boston directly or inversely proportional to the speed?
   1. Explain your reasoning and include a formula for the proportion. Answers will vary. It is an inverse proportion because as the driving speed goes up, the time it takes to travel the distance goes down. We have a constant product. Distance = (Rate) (Time)
   2. To get to Boston from Waterbury in 2 hours how fast would you have to drive? 75.6 mph
   3. Is it reasonable to think you can drive to Boston from Waterbury in 2 hours? Hopefully NO since you would be exceeding the speed limit the whole time.
2. Consider the functions .
   1. What are their domains? [0, for both
   2. What are their ranges? [0, for both
   3. Using a window that allows you to study their graphical behavior compare and contrast them with regard to concavity, increasing/decreasing behavior, x-and y-intercepts.

Both are increasing over their domain but f is concave down while g is concave up. The y-intercept is (0,0) for both of them and they have the same x-intercept (0,0) as well.

1. Suppose an astronaut weighs 60 kilograms when standing on the earth. When she flies far away from the earth, her weight changes because the effect of the gravity of the earth is decreased. The function describes the weight of an astronaut who weighs 60 kilograms on earth as a function of h, the astronaut’s height above the earth in kilometers.

a. The International Space Station orbits at a height about 400 kilometers above the earth. Find the weight of the astronaut while she is on the International Space Station.  
53.1 kilograms  
  
b. What is the domain of W(h)?  
[0, ∞)  
  
c. If the domain is restricted as in question (b), would W(h) have an inverse? Explain why or why not. If you think W(h) does have an inverse, find an equation for the inverse function.  
If the function is restricted as in (b), the restricted function is 1-1 so W(h) does have an inverse:   
  
d. Is it ever possible that the astronaut will weigh more than 60 kilograms? Explain why or why not.  
The function decreases when h > 0 so W(h) can never exceed 60 kilograms  
  
e. Find the height the astronaut would have to reach in order to weigh half as much as she does on earth.  
 = 30; h = 2651 kilometers

**Units 2 – 5**

1. Solve the following equations over the set of real numbers. Watch out for extraneous roots. Give answers in exact form.
   1. 9 (1 is extraneous)
   2. -5/2
   3. log12/log1.7
   5. +
   6. no real roots
   7. + no solutions
   8. 0 is a solution and -6 does not check
   9. 8log x = 100log 4 – 2 log x 410

**Unit 5**

1. Rewrite each equation as a logarithmic equation.

4. = 1
5. Rewrite each equation as an exponential equation.
   1. 102 = 100
   2. 34 = 81
   3. = -1 5-1 = 0.2
6. Find an equivalent expression for each expression using properties of logarithms. Your new expressions may not appear to be “simpler.”
   1. log (2x) – log(x + 5) =
   2. log (x5) = 5 log x
   3. log(5x + 4) + log(x – 3) – (log (x2- 9) =
7. Your grandmother when she passed gave you $10,000 to invest to pay for postsecondary education. You can place your investment in bonds that guarantee 5.6% interest compounded continuously or you can open a CD that guarantees 5.7% interest compounded monthly for 5 years. Compute the value of each investment for 5 years. Which investment will earn you more interest over the 5 year period and how much more interest will you earn?

$13,231.30 vs $13,288.65 so the CD earned more $57.35.

1. How long will it take you to double $5000 in a bank guaranteeing 5% interest compounded continually? Does the time depend upon how much money you have in your account? Explain.

Almost 14 years or 13.86 years

1. Why isn’t the log(a/b) = (log a)/(log b)? By counterexample, log(100/10) = log 10 = 1. But log 100 / log 10 = 2/1 = 2. For a property to be true it must always work.
2. During your study of the exponential family in algebra 1 and this year in algebra 2 you have met the doubling penny problem in varying scenarios. If your uncle gives you 2cents on the first of March, 4 cents on the 2nd of March, 8 cents on the 3rd of March, what is the total amount of money he will have given you by the end of the month. Include the March 31 payment. $42,949,672.96
3. Logs are stacked in layers where each succeeding layer has one less log than the row below it. If the bottom layer has 16 logs and the top layer has 7 logs, how many logs are in the pile? Use a formula to find the sum. Hint: this is not a geometric series. 5 times 23 or (10/2)(16 + 7) = 115
4. Suppose you decide to practice for a major track and field event. You decide to start by running each morning but you will decrease your running each day so you can increase your other conditioning work. You run 2 miles the first day, 2/3 mile on day 2, 2/9 on day 3, 2/27mile on day 4. How many miles will you have run in the first week? In the first 2 weeks? After many, many weeks? 2.9986, 2.999999, 3 miles
5. Davon decided that even though he was very young he would start saving for his retirement. He is only 25 but decided he would put $100 dollars in an investment at the end of each month and would do this for 30 years. he knows he can at least get 5% interest compounded monthly. When he was at a family Thanksgiving dinner when he was 30years old his brothers and sisters were teasing him but his dad was very impressed with his plan to retire comfortably.
   1. Compute how much money he will have invested by the time he is 55 so he can convince his siblings to start saving. $83,225.86
   2. Davon then plans to take the amount he has in part a and invest it at at least 6% compounded continuously until he is 67. By the end of the 12 additional years how much money will he have? $170,981.97
   3. Davon decided that he wanted more money when he retired and he now had had several raises so he decided to see what the difference would be if he saved $200 each month till he was 55. So he already had saved 100 dollars for 5 years and now will save 200 for the remaining 25 years. How much will he have saved by the time he is 55? $6800.61 + 119101.94 = $125,902.55
   4. Davon then of course will take the money form part c and invest it at at least 6% compounded continuously until he is 67. By the end of the 12 additional years how much money will he have? $258,658.38
   5. How much of the total in part d was interest earned over the 42 years? $258,658.38 – 66000 = $192658.38
6. Cesium-137 is a dangerous radioactive element that can be created when a nuclear power plant has a meltdown, as did the Fukushima Daichi plant in Japan in 2011. The half-life of Cesium-137 is approximately 30 years.

a. Find a function that describes the amount A(y) of Cesium-137 remaining after *y* years after starting with an initial amount of *a*.  
A(y) = a(.977)y  
  
b. Your function in question (a) should be in the form of an exponential function. Give an interpretation of what the base of the exponential function in question (a) means.  
The base is .977; this number means that every year, the proportion of Cesium-137 that decays is 1 – 0.977 = 0.023, or 2.3%

1. The half-life of iodine-131 is eight days. Hospitals use it in diagnosing thyroid gland conditions. How much of a two-gram sample will remain after 6 days. Hint: find k first.

k = .0866 approximately

1.19 grams

1. Stamford is the fastest growing large city in Connecticut and is one of the fastest growing towns of any size in the state. The population of Stamford in 2010, according to the U.S. Census, was 122,643. By 2011, the population had grown to 124,010.
2. What is the amount of the increase in population in Stamford from 2010 to 2011? What is the percentage increase in population from 2010 to 2011?

The amount of the increase is 124010 – 122643 = 1367. Since , the percentage increase in population is about 1.11%.

1. Suppose that the population continues to grow linearly. Find a function that gives the population P(t) as a function of t, the number of years since 2010.

P(t) = 122643 + 1367t

1. Suppose that the population continues to grow exponentially. Find a function that gives the population P(t) as a function of t, the number of years since 2010.

P(t) = 122643(1.0111)t

1. Using your two functions, find the expected population of Stamford in 2050 if the population grows (i) linearly, and (ii) exponentially.

Using the linear function, P(40) = 177,323. Using the exponential function, P(40) = 190,724.

**Unit 6**

1. The average daily high temperature in Hartford, CT can be approximated by the following function: , where d is the number of days since the beginning of the year and T(d) is the average daily high temperature on that day. You may want to graph the function on your grapher.
2. What is the period of this function? Why?

The period is 365; this is the number of days in a year

1. Find the approximate average daily high temperature in Hartford on April 1.

April 1 is approximately 90 days after the start of the year; T(90) = 52.8

1. What is the approximate highest average daily high temperature in Hartford? On approximately what day of the year does that temperature occur?

From the graph, the maximum temperature is about 84 degrees, which occurs on about Day 195 of the year, or around July 15.

1. What is the approximate lowest average daily high temperature in Hartford? On approximately what day of the year does that temperature occur?

From the graph, the minimum temperature is about 34 degrees, which occurs on about Day 15 of the year, or around January 15.

1. The tides at a Connecticut beach are cyclical. On a given day, low tide occurs at midnight and is 5 feet. High tide occurs at noon and is 11 feet. Find a mathematical model. Over one day, the points (0, 5), (12, 11), (24, 5) can be used to determine a model. The equation of the midline is y = 8 and the amplitude is 3. Starts at a low so f(x) = -3cos((π/12) x) + 8.
2. A chipmunk population varies with the seasons of a year. In the winter when it is cold and food is scare it reaches a low and in the summer when food is available and new litters have been born it reaches a high. Suppose in a particular forest area the population reaches a low of 200 chipmunks in January and a high in July of 500. Model the chipmunk population of this forest area. (0, 200),(6, 500) . A = (500 – 200) /2 ,midline y = 350 so f(x) = -150 cos (πx/6) + 350
3. Suppose Electrical current alternates between 60 V and 120 V and back every 0.4 seconds. Assume the initial voltage is 120 V. Determine a modeling function. (0, 120), (.2, 60), (.4, 120) The midline y = 90, A = 30 so g(x) = 90 + 30 cos(5πx)
4. Is the relationship between arc length and the central angle measure of a circle a direct or inverse proportion? Direct Explain. s = r ϴ where r is the constant of proportionality fits the model y = kx. r is constant for the circle and ϴ and s vary.
5. Write an equation that models each description.
   1. A sine function that has been vertically stretched by a factor of 2 and shifted down 4 units. y = 2 sin(x) – 4
   2. A cosine function that has been horizontally stretched by a factor of 0.5 and shifted up 2 units. y = cos(0.5x) + 2
   3. For the function in part a, what is the amplitude, period and equation of the midline?

A = 2, P = 2π, y = -4

* 1. For the function in part a, what is the amplitude, period and equation of the midline?

A = 1,P = 4π, y = 2

1. Explain why an angle that measures 1 radian is about 57 degrees when measured in degrees.

Answers will vary but should include that a radian subtends an arc whose measure is one radius length and about 6.28 radii fit along the circumference of a circle so that 6.28 radians are about 360 degrees which means one radian is about 57 degrees.

1. Find 4 coterminal angles for each given angle. Two must be positive and 2 negative.
   1. 30ᵒ 390ᵒ , 750ᵒ, -330ᵒ, -690ᵒ
   2. -45ᵒ 315ᵒ, 675ᵒ, -405ᵒ, -765ᵒ
   3. 3π/4 11π/4, 19π/4, -5π/4, -13π/4
   4. 7π/6 19π/6, 31π/6, -5π/6,-17π/6
2. a. Explain why x2 + y2 = 1 does not define a function.

Many answers are possible, such as this is the equation of the unit circle and its graph does not pass the vertical line test, or both (0, -1 and (0, 1) are y-intercepts.

b. What is the domain of this relation? [-1, 1]

c. What is the range? [-1, 1]

**Units 1, 2, 5**

1. Use the tables below to determine whether each function is linear, quadratic, exponential, or none of these. Explain your answers. Only complete the columns needed to make your determination.

A.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Input**  ***x*** | **Output**  ***f (x)*** | **Difference between outputs** | **Difference between differences** | **Ratio of outputs** |
| 1 | 5 |  |  |  |
| 2 | 12 | 7 |  |  |
| 3 | 21 | 9 | 2 |  |
| 4 | 32 | 11 | 2 |  |
| 5 | 45 | 13 | 2 |  |
| 6 | 60 | 15 | 2 |  |

Quadratic y = x2 + 4x. In some STEM classes you may have had students note that when Δ x = 1, a = .5 ΔΔy so feel free to ask for an a value. a = 1

B.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Input**  ***x*** | **Output**  ***f (x)*** | **Difference between outputs** | **Difference between differences** | **Ratio of outputs** |
| 1 | 1.9 |  |  |  |
| 2 | -0.2 | -2.1 |  |  |
| 3 | -2.3 | -2.1 |  |  |
| 4 | -4.4 | -2.1 |  |  |
| 5 | -6.5 | -2.1 |  |  |
| 6 | -8.6 | -2.1 |  |  |

Linear y = -2.1 x + 4

C.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Input**  ***x*** | **Output**  ***f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference between differences** |
| 1 | 12 |  |  |  |
| 2 | 48 | 36 |  |  |
| 3 | 192 | 144 | 4 |  |
| 4 | 768 | 576 | 4 |  |
| 5 | 3072 | 2304 | 4 |  |
| 6 | 12288 | 9216 | 4 |  |

Exponential y = 3(4^x)

D. If a table above describes a linear function, find an equation for the function.

y = -2.1x + 4

E. If a table above describes an exponential function find an equation for the function.

y = 3(4^x)

**Units 1 – 6**

1. On the axes provided, sketch a graph of the following functions and answer the related questions:





1. What is the y-intercept? (0, 6)
2. What are the x-intercepts? Find the exact values, no decimal approximations.

2 +√2 and 2- √2

1. What are the coordinates of the vertex? (2, -6)
2. Why without graphing, did you know the parabola would open up? a is 3 > 0
3. What is the domain of this function? (
4. Does this function have a maximum or minimum value? What is it?

Minimum value of -6





1. What is the y-intercept? (0,3)
2. What are the x-intercepts? Find the exact values, no decimal approximations.

(-7/2, 0), (3/2,0)

1. What are the coordinates of the vertex?

(-1,5)

1. What is the domain of this function? (
2. Does this function have a maximum or minimum value? maximum What is it? 5





1. What is the y-intercept? (0, 9)
2. What are the x-intercepts? Find the exact values, no decimal approximations.

(-3, 0), (-1, 0)

1. What is the domain of this function? (
2. Does this function have a maximum or minimum value? What is it? If not, explain why it does not?

No, the range of the function is (.

1. Describe the end behavior. As x
2. Where is f increasing? (The -1.75 is an approximation.





1. What is the y-intercept? -5/9
2. What are the x-intercepts? Find the exact values, no decimal approximations. -5/2
3. If there is a horizontal asymptote, draw it and write down its equation.

Y = 2/3

1. If there is a vertical asymptote, draw it and write down its equation. x = 3
2. What is the domain of this function?
3. Describe the end behavior. As x
4. Where is r decreasing?
5. ) - 1





1. What is the y-intercept? (0, -0.5)
2. What are the x-intercepts? Find the exact values, no decimal approximations.

log 2/(log 3)

1. If there is a horizontal asymptote, draw it and write down its equation. Otherwise write none.

y = -1

1. If there is a vertical asymptote, draw it and write down its equation. Otherwise write none.

None

1. What is the domain of this function? (
2. on [0, 2]





1. What is the y-intercept? (0, 6)
2. What are the x-intercepts? Find the exact values, no decimal approximations.

(π/2), (3π/2)

1. What is the equation of the midline? y = 3
2. What is the amplitude? 3
3. What is the period? π
4. What is the domain of this function [0, 2π]

**Unit 7**

**7.1**

1. What is the relationship between the amount of sleep that college students get each night and the amount of hours they study per day? The following data shows the typical number of hours of sleep per night (*x*) and the typical number of hours of studying per day (*y*) for a random sample of ten college students. The data and scatterplot are shown below.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  | | --- | --- | | Hours of Sleep | Hours of Studying | | 4 | 8 | | 4 | 5 | | 5 | 6 | | 6 | 5 | | 6 | 4 | | 7 | 3 | | 8 | 5 | | 9 | 4 | | 10 | 5 | | 10 | 2 | |  |

1. Calculate the sample correlation coefficient.

1. What does the sample correlation coefficient indicate about the sample?

There is a negative correlation (or negative relationship) between the two variables.

1. Randomization samples were obtained from the original sample under the assumption that there is no relationship (no correlation) between the two variables. Sample correlation coefficients from 100 randomization samples are plotted on the dot plot below.



According to the randomization distribution, what is the *P*-value of the observed correlation coefficient?

There are 2 sample proportions less than or equal to -0.63, so the *P*-value is 2/100 = 2%.

1. Is there sufficient evidence to conclude that a correlation exists between the amount of sleep (per night) and the amount of studying (per day) for all college students?

The *P*-value is less than 5%, so we can reject the assumption that there is no correlation between the two variables. We can conclude that a correlation exists between the amount of sleep (per night) and the amount of studying (per day) for all college students.

**7.2**

1. Read the description of the study below and answer the questions that follow.

In April 2016 the Gallup organization surveyed 1,015 US adults via phone interviews. The survey asked respondents whether or not they feel upper-income people are paying their fair share, too much, or too little, in federal taxes. Of all the adults who responded, 21% said “fair share”, 15% said “too much”, and 61% said “too little”. Among adults who indicated that they are Democrats, 14% said “fair share”, 9% said “too much”, and 75% said “too little”. Among adults who indicated that they are Republicans, 32% said “fair share”, 20% said “too much”, and 45% said “too little”.

1. What type of study was performed?

Sample survey or observational study

1. What is the population of interest in this study?

The population of interest is all US adults.

1. What reasonable conclusions can we draw from this study?

Answers will vary. Acceptable answers should contain a statement about the population of U.S. adults, the population of adult Democrats in the U.S., or the population of adult Republicans in the U.S. Examples include: the majority of U.S. adults believe that upper income people are paying too little federal taxes, or the majority of adults Democrats in the U.S. believe that upper income people are paying too little federal taxes.

1. The data set below contains the number of hits given up by five major-league baseball (MLB) starting pitchers during the 2015 regular season. Find the mean absolute deviation (MAD) and standard deviation (SD) of this sample data. Interpret one of these measures.

|  |  |
| --- | --- |
| Pitcher | Hits |
| Felix Hernandez (SEA) | 180 |
| David Price (TOR) | 190 |
| Alfredo Simon (DET) | 201 |
| Scott Kazmir (HOU) | 162 |
| Jimmy Nelson (MIL) | 163 |

The sample mean is .

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 162 | -17.2 | 295.84 |
| 163 | -16.2 | 262.44 |
| 180 | 0.8 | 0.64 |
| 190 | 10.8 | 116.64 |
| 201 | 21.8 | 475.24 |

The mean absolute deviation is (17.2 + 16.2 + 0.8 + 10.8 + 21.8) / 5 = 66.8 / 5 = 13.36.

The sample standard deviation is:

The MAD indicates that the typical deviation from the sample mean is 13.36 hits. The sample standard deviation indicates that the typical deviation from the sample mean is 16.96 hits.

1. The times it takes male students at a large university to run a mile are approximately normally distributed, with a mean of minutes and standard deviation of minutes.
2. Use the Empirical Rule to find the interval that contains the middle 68% of 1-mile run times.

1. Use the Empirical Rule to find the interval that contains the middle 95% of 1-mile run times.

1. A male student at the university has a 1-mile run time of 8 .1 minutes. Find and interpret the *z*-score of this 1-mile run time.

This student’s 1-mile run time is 1.04 standard deviations above the mean of male students’ 1-mile run times.

1. What 1-mile run times in this distribution are unusual?

1-mile run times outside the middle 95% interval are considered unusual. These are times less than 5.61 minutes and greater than 8.89 minutes.

**7.3**

1. In a random sample of *n* = 40 high school seniors, 18 say that they play sports in the afternoon or evening during the week. Determine the 95% confidence interval for the proportion of all high school seniors who play sports in the afternoon or evening during the week.
2. What is the population parameter of interest in this problem? What is the sample statistic?

The population parameter is the proportion of all high school seniors who play sports in the afternoon or evening during the week. The sample statistic is the sample proportion, 18/40 = 0.45.

1. Determine the point estimate.

The point estimate is the sample proportion 0.45.

1. Calculate the margin of error for the 95% confidence interval.
2. Construct and interpret the 95% confidence interval.

. We are 95% confident that the proportion of all high school seniors who play sports in the afternoon or evening during the week is between 29.3% and 60.7%.

1. A researcher claims that less than 40% of high school students plan to start their own business. To test this claim you obtain a random sample of 60 high school students and ask each of them whether they plan to start their own business. Of the 60 students surveyed, 18 say yes. Does this sample provide sufficient evidence to support the researcher’s claim?
2. In this hypothesis test, what is the assumed value of the population proportion?

The assumed value of the population proportion is 0.40.

1. Calculate the sample proportion, .
2. The dotplot below displays a randomization distribution of 100 sample proportions based on the assumption that the population proportion is 0.40.



According to the randomization distribution, what is the *P*-value for the observed sample proportion?

There are 4 sample proportions less than or equal to 0.30, so the *P*-value is 4/100 = 4%.

1. Is there sufficient evidence to conclude that the proportion of high school students who plan to start their own business is less than 40? Explain.

The *P*-value is less than 4%, so we can reject the assumption that the population proportion is 0.40. There is sufficient evidence to conclude that the proportion of high school students who plan to start their own business is less than 40%.

1. The Gallup Student Poll asks high school students if they feel that they can find many ways around problems. Suppose this question was posed to high school students and the following results emerged. In a random sample of 40 female high school students, 34 said yes. In a random sample of 40 male high school students, 28 said yes. Do these samples provide sufficient evidence that the proportion of all female high school students who feel that they can find many ways around problems is greater than the proportion of male high school students who feel the same way?
2. State the assumption about the population proportions that we will test.

We assume that the population proportions are equal, so the proportion of female high school students and the proportion of male high school students who feel that they can find many ways around problems are the same.

1. Let represent the sample proportion for female high school students. Let represent the sample proportion for male high school students. Find the difference in the observed sample proportions, .
2. The dotplot below displays a randomization distribution of 100 differences in sample proportions, , assuming the population proportions are equal.



Use this distribution to estimate the *P*-value for the observed difference in sample proportions.

There are 4 sample proportions greater than or equal to 0.15, so the *P*-value is 4/100 = 4%.

1. Is there sufficient evidence to conclude that the proportion of all female high school students who feel that they can find many ways around problems is greater than the proportion of male high school students who feel the same way? Explain.

The *P*-value is less than 5%, so we can reject the assumption that the population proportions are equal. There is sufficient evidence to conclude that the proportion of female high school students who believe that they can find many ways around problems is greater than the proportion of male high school students who feel the same way.

**7.4**

1. A random sample of *n* = 50 high school students reported the amount of time that they spend watching television on weekdays during the school year. The sample has a mean of minutes and sample standard deviation of 18.5 minutes. Use these sample statistics to construct a 95% confidence interval for the mean amount of time that all high school students spend watching television on weekdays during the school year.
2. What is the population parameter of interest in this problem? What are the sample statistics?

The population parameter is the mean amount of time that all high school students spend watching television on weekdays during the school year. The sample statistics are the sample mean (110 minutes) and the sample standard deviation (18.5 minutes).

1. Determine the point estimate.

The sample mean .

1. Calculate the margin of error for the 95% confidence interval.
2. Construct and interpret the 95% confidence interval.

. We are 95% confident that the mean amount of time that all high school students spend watching television on weekdays during the school year is between 104.8 and 115.2 minutes.

1. A student claims that the average number of hours students at her high school spend on the computer each week is less than 30. To test this claim, suppose you obtain a random sample of *n* = 10 students from the high school and ask each of them how many hours they spend on the computer each week. The results of the survey are shown below.

Number of hours on the computer each week

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 40 | 35 | 10 | 15 | 20 | 10 | 40 | 10 | 20 | 50 |

Does this sample provide sufficient evidence to support the student’s claim?

1. State the assumption about the population mean that we will test.

We assume that the population mean, the mean amount of time students at the high school spend on the computer each week, is equal to 30 hours.

1. Calculate the sample mean, .

1. The histogram below displays a randomization distribution of 100 sample means assuming the population mean is 30 hours per week.



Use the distribution to estimate the *P*-value for the observed sample mean.

There are 19 sample means that are less than or equal to 25, so the *P*-value is 19/100 = 19%.

1. Is there sufficient evidence to conclude that the average amount of time students at the school spend talking on the phone is less than 12 hours per week? Explain.

The *P*-value is greater than 5%, so we cannot reject the assumption that the population mean is 30 hours per week. There is not sufficient evidence to conclude that the mean amount of time students at a high school spend on the computer each week is less than 30 hours per week.

1. Do male and female high school students have differing views on the importance of reducing pollution? A researcher was interested in investigating this question at a local school. Using the *Census at Schools* questionnaire, he asked students to rate their view on the importance of reducing pollution from 0 to 1000. 0 indicates “not important” and 1000 indicates “very important”. He obtained data from a random sample of 25 female students and a random sample of 25 male students. The sample statistics are shown below.

|  |  |  |
| --- | --- | --- |
| **Female Students** |  | **Male Students** |
|  |  |  |
|  |  |  |

Do these samples provide sufficient evidence to conclude that female students at the school find that reducing pollution is more important than male students at the school?

1. State the assumption about the population means that we will test.

We assume that the population means are equal, so the mean rating for female students at the school is equal to the mean rating for male students at the school.

1. Let represent the mean rating for the sample of female students. Let represent the mean rating for the sample of male students. Find the difference in the observed sample means, .
2. The histogram below displays a randomization distribution of 100 differences in sample means assuming there is no difference between the population means.



Use this distribution to estimate the *P*-value for the observed difference in sample means.

There are about 11 differences in sample means that are greater than or equal to 120, so the *P*-value is 11/100 = 11%.

1. Is there sufficient evidence to conclude that female students at the high school feel that reducing pollution is more important than male students at the high school? In other words, is there sufficient evidence to conclude that the mean rating on the importance of reducing pollution of all female students is greater than the mean rating of all male students?

The *P*-value is greater than 5%, so we cannot reject the assumption that the population means are the same. There is not sufficient evidence to conclude that the mean rating on the importance of reducing pollution of all female students is greater than the mean rating of all male students.

**7.5**

1. Gallup reports that 42% of students in grades 5 – 12 in the U.S. say that they are learning financial literacy skills – skills regarding how to earn and save money for the future. Let’s assume that 0.42 is the population proportion – the proportion of all students in grades 5 – 12 in the U.S. who say they are learning financial literacy skills. Suppose we randomly select six students in grades 5 – 12 in the U.S.
2. What is the probability that exactly three students will say that they are learning financial literacy skills.
3. What is the probability that five or more students will say that they are learning financial literacy skills.
4. Construct a histogram to show the probabilities in the binomial probability distribution. Identify any unusual outcomes of the binomial random variable.



Answers may vary. The outcome of 6 is unusual since its probability is less than 1%.

1. The heights of adult women in the U.S. are approximately normally distributed with a mean of 65.5 inches and a standard deviation of 2.5 inches.
2. Model this distribution on the normal curve below. Indicate the positions of the mean and the values that are one, two and three standard deviations away from the mean.





1. Suppose an adult woman in the U.S. has a height of 70 inches. What is the *z*-score for this height?
2. Find the probability that a randomly selected adult woman in the U.S. has a height greater than 70 inches. Shade in the corresponding area on the normal curve above.

0.036. The area is shown in the answer above.

1. What percent of adult woman in the U.S have heights between 60 and 65 inches? Shade in the corresponding area on the normal curve above.

40.7%. The area is shown in the answer above.

1. Would it be unusual to randomly find an adult woman in the U.S. with a height of 61 inches? Explain.

The height of 61 inches has a z-score of -1.8. Since this z-score is not less than or equal to -2, or not greater than or equal to 2, this result would not be unusual.

1. Find the probabilities indicated in the normal curves below. The normal curves model SAT mathematics scores for a population of high school students. The mean and standard deviation of the normal distributions are indicated below.



Probability = 0.152



Probability = 0.044

1. A student claims that the mean time it takes students at his school to travel to school is less than 15 minutes. To test this claim we obtain a random sample of *n* = 32 adults and obtain their travel time to school. The random sample has a mean commute time of 13.7 minutes. Assume students’ travel times to this school are normally distributed with standard deviation minutes. Does this sample provide evidence that the population mean is less than 15 minutes?
2. State the assumption about the population mean that we will test.

We assume that the population mean, the mean travel time to school for students at the school, is 15 minutes.

1. Determine the mean and standard deviation of the sampling distribution of sample means for random samples of size *n* = 32. Round the standard deviation to the nearest tenth.

Mean of sample means = 15

Standard deviation of sample means =

1. Sketch the sampling distribution of sample means below. Indicate the positions of the mean and the values that are one, two and three standard deviations away from the mean.





1. Is the observed sample mean statistically significant? Explain.

Yes, the probability of observing a sample mean less than or equal to 13.7 is 0.015 = 1.5%. This *P*-value is less than 5%.

1. What can we conclude about the claim that the mean travel time to school for students at the school is less than 15 minutes?

We can reject the assumption that the mean travel time to school for students at the school is 15 minutes and we can support the claim that the mean travel time is less than 15 minutes.

**Unit 8.1**

1. Consider the matrices *A* = , *B* = and .
2. Calculate .
3. Calculate .
4. Compare the two results and discuss in terms of the properties of matrices.

The two results are the same because matrix multiplication is associative.

1. Does your finding apply to any two matrices?

Not all matrices can be multiplied. The associative property applies to matrices that have the correct number of rows and columns so that they may be multiplied. For two matrices *A* and *B*, the product matrix *AB* is defined only if the number of rows of *B* matches the number of columns of *A.*

**8.1**

**(Note: Question 75 is a replacement for question 17 above.)**

1. a. Is the operation of addition for the set of real numbers commutative? Yes Provide one concrete example. 4 + 7 = 7 + 4

b. Is the operation of subtraction for the set of real numbers commutative? No Justify your answer. 10 – 4 = 4 – 10 is not true

c. Is matrix multiplication for 2X2 matrices commutative? NO Justify your answer.

= A counter example is all that is needed

=

= =

**Unit 8.3**

1. Consider the matrices *A* = and the matrix
   1. Find the matrix

* 1. Based on your answer above, what can you say about matrices *A* and *B*?

You cannot claim that they are inverse matrices. Two matrices are inverses only when

. You would need to check the product matrix *BA*

* 1. Calculate
  2. Compare the two results and discuss in terms of the properties of matrices. The two results are different which verifies that matrix multiplication is not commutative. It also shows that nonsquare matrices do not have inverses, because the product *AB* cannot be the same size as product *BA*, so they cannot be equal matrices.

**Unit 8.2 & 8.4**

1. Consider the matrices *A* = and the vector .
2. Multiplying the vector by the matrix *A* transforms the vector to a new vector
3. Find a matrix *B* so that multiplying the vector by *B* will undo the transformation.
4. Multiply by *B* to check that the matrix *B* you found reverses the effect of the transformation.