**Activity 5.1.8B (+) Consequences of Being Inverse Functions**

In this activity, we will use your study of inverse functions and composition of functions from unit 1 to verify two important properties of exponents and logarithms. We will show that

$log\_{b}b^{x}=x$ and $b^{log\_{b}x}=x,$

from the composition of the two functions $f\left(x\right)= b^{x}$ and $g\left(x\right)=log\_{b}x$. First let us verify for base 10. **For the moment we will not assume the two functions are inverse functions.**

1. Study the completed rows and fill in the two empty rows and the last entry of column 4.

|  |  |  |  |
| --- | --- | --- | --- |
| x | f(x)=10x | x ( column 2 outputs of f) will be the inputs for g(x)= log x | g(x) =log x, the exponent |
| -1 | 0.1 | 0.1 | -1 |
| 0 | 1 | 1 | 0 |
| 1 | 10 | 10 | 1 |
| 2 | 100 | 100 | 2 |
| 3 | 1000 | 1000 | 3 |
| 5 |  |  |  |
| 6 |  |  |  |
| a | 10a | 10a |  |

1. Let $f\left(x\right)= 10^{x}$ and $g\left(x\right)=log\_{10}x$.
2. Find $g\left(f\left(x\right)\right).$ g(10x) = log10 10x= x log10 10 = \_\_\_\_\_\_\_\_\_
3. Explain the steps in part a. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. For what values $x$ does this equation hold? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
5. Let $f\left(x\right)= b^{x}$ and $g\left(x\right)=log\_{b}x$.
6. Find $g\left(f\left(x\right)\right)$ = $g\left(b^{x}\right)=log\_{b}b^{x}=\\_\\_\\_\\_\\_\\_\\_\\_$ = \_\_\_\_\_\_\_\_\_\_\_\_\_
7. Explain the steps in part a. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
8. For what values $x$ does this equation hold? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
9. Simplify the following
10. $log\_{3}3^{2}=$
11. $log\_{2}2^{5}$ =
12. $log\_{4}\frac{1}{16}=$
13. $log\_{3}27=$
14. $log\_{10}10000000=$
15. $log\_{10}\frac{1}{10}=$
16. Complete the table.

|  |  |  |  |
| --- | --- | --- | --- |
| x | g(x)=log x | x (the outputs of g from column 2) will be the inputs for f | f(x)= 10x |
| -10 |  |  |  |
| 0 |  |  |  |
| 10 | 10 | 1 | 10 |
| 100 | 100 | 2 | 100 |
| 1000 | 1000 | 3 | 1000 |
| 10000 |  |  |  |
| 189999 |  |  |  |
| a | log a | log a |  |

1. Let $f\left(x\right)= 10^{x}$ and $g\left(x\right)=log\_{10}x$.
2. Find $f\left(g\left(x\right)\right).$
3. Why does $f\left(g\left(x\right)\right)=x$? Explain the steps you need to take in part a.
4. For what values $x$ does this equation hold?
5. Let $f\left(x\right)= b^{x}$ and $g\left(x\right)=log\_{b}x$.
6. Find $f\left(g\left(x\right)\right).$
7. Why does $f\left(g\left(x\right)\right)=x$?
8. For what values $x$ does this equation hold?
9. Simplify the following
10. $2^{log\_{2}x}=$
11. $10^{log\_{10}x}=$
12. $\frac{1}{2}^{log\_{\frac{1}{2}}x}$=

You have verified two important properties of exponents and logarithms,

$log\_{b}b^{x}=x$ **and** $b^{log\_{b}x}=x.$

The logarithm function undoes the exponential function and the exponential undoes the logarithm function (for the same base of course).You will need to use these properties in future exercises.

Because these two functions are inverses of each other let us stress that these functions undo each other . That is, if (a, b) is on the graph of f then (b, a) is on the graph of f -1. So if we input a into the formula for f and get the output b and then use b as an input for f -1 the output must be a

1. In activity1.6.4 you determined if pairs of functions were inverses using function composition. You have just demonstrated that f(*x*) =logb*x* and g(*x*)=b*x* are inverses.

Determine if the following pairs of functions are inverses using function composition.

1. f(x) = x3 – 3 and g(x) = $\sqrt[3]{x+3}$
2. g(x) = -0.5 x +4 and k(x)= -2x + 10
3. f(x) = $\frac{1}{3x}$ and t(x) = $\frac{1}{3x}$