**Activity 4.3.1 Graphing Rational Functions I**

In this activity, you will be learning some new vocabulary**.** In investigation 4.2, we saw that a rational function can be expressed as the quotient of two polynomials. Because we are dealing with a quotient, as with fractions, we have one big concern. What is it? \_\_\_\_\_\_\_\_\_\_\_ When the polynomial in the denominator of the rational expression is linear, the equation of the rational function can be written as

**.**

In this activity and the next two, you will be discovering what a vertical asymptote (VA) and a horizontal asymptote (HA) are.Over the course of the activities you will develop definitions for these important lines**.** At this point, we can say that a vertical asymptote is a vertical line and we write the equation of a VA as “x = a constant”. A horizontal asymptote (HA) is a horizontal line and we write the equation of a HA as “y = a constant”. Later in the activity and in the next two activities we will get more specific regarding the definitions of a VA and HA.

The form above may look familiar to you as we saw something similar when working with the vertex form of a quadratic, . We also saw this form when working with the vertex form of the absolute value function, .

1. How did the *a*, *h* and *k* parameters affect the graph of the quadratic and absolute value functions?

2. Do you think that *a, h* and *k* will have similar effects on the graph of the rational function? Explain.

Let’s start with the basic graph ofwhich you met in Algebra I, Unit 1 of Algebra II and Investigation 4.2. If we enter this into the Y1= on our graphing calculator and graph we will see a graph and a table of values like the ones shown in the figures below. This kind of graph is called a hyperbola.

 

1. Notice the error in the table when x = 0. What is happening at this point on the graph? \_\_\_\_\_\_\_\_\_ What happens when you evaluate the function at 0? \_\_\_\_\_\_\_ What does this mean about the domain of the function?
2. For the basic graph of, the vertical asymptote is the vertical line with equation x = 0. The horizontal asymptote is the horizontal line with equation y = 0. You will not “see” these lines on the calculator but we can look at the behavior of the graph in order to “see” them. The VA and HA are shown with **dashed lines** in the graph below.
   1. Complete the table of values for this function.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | -1,000 | -100 | -10 | -1 | 1 | 10 | 100 | 1,000 |
| y |  |  |  |  |  |  |  |  |

* 1. What is happening to the y-values as the x-values become really large and really small? Why do you think this is happening?





* 1. For our function,  (shown above), what are the *a, h* and *k* values?

a= \_\_\_\_\_\_, h= \_\_\_\_\_\_, k = \_\_\_\_\_\_

VA: \_\_\_\_\_\_\_\_\_HA: \_\_\_\_\_\_\_\_\_\_\_\_\_

* 1. What is the domain? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  2. What value is not included in the domain? Why?
  3. How would you describe the behavior of the function near x = 0, the vertical asymptote?
  4. How would you describe the end behavior?

1. Now, let’s look at how our graph is transformed as we vary the parameters of *h* and *k*.

Enter the function  into Y1 on your calculator. Record the values for *a, h* and *k* below.

1. a= \_\_\_\_\_\_, h= \_\_\_\_\_\_, k = \_\_\_\_\_\_



1. How did the graph of the function shift? Describe the transformation.
2. What is the equation of the VA?\_\_\_\_\_\_\_ What is the equation of the HA? \_\_\_\_\_\_\_\_

1. How are the VA and the HA related to the *h* and *k* values of your function?
2. What is the domain? What value is not included in the domain? Why?
3. How would you describe the behavior of the function near x = 0, the vertical asymptote?
4. How would you describe the end behavior?
5. Enter the function  into Y1 on your calculator. Record the values for *a, h* and *k* below.
6. a= \_\_\_\_\_\_, h= \_\_\_\_\_\_, k = \_\_\_\_\_\_
7. How did the graph of the function shift? Describe the transformation.
8. What is the equation of the VA?\_\_\_\_\_\_\_ What is the equation of the HA? \_\_\_\_\_\_\_\_

1. How are the VA and the HA related to the *h* and *k* values of your function?
2. What is the domain? What value is not included in the domain? Why?
3. How would you describe the behavior of the function near x = 0, the vertical asymptote?
4. How would you describe the end behavior?
5. Enter the function  into Y1 on your calculator. Record the values for *a, h* and *k* below.
6. a= \_\_\_\_\_\_, h= \_\_\_\_\_\_, k = \_\_\_\_\_\_



1. How did the graph of the function shift? Describe the transformation.
2. What is the equation of the VA?\_\_\_\_\_\_\_ What is the equation of the HA? \_\_\_\_\_\_\_\_

1. How are the VA and the HA related to the *h* and *k* values of your function?
2. What is the domain? What value is not included in the domain? Why?
3. How would you describe the behavior of the function near x = 0, the vertical asymptote?
4. How would you describe the end behavior?
5. Enter the function  into Y1 on your calculator. Record the values for *a, h* and *k* below.

1. a= \_\_\_\_\_\_, h= \_\_\_\_\_\_, k = \_\_\_\_\_\_
2. How did the graph of the function shift? Describe the transformation.
3. What is the equation of the VA?\_\_\_\_\_\_\_ What is the equation of the HA? \_\_\_\_\_\_\_\_

1. How are the VA and the HA related to the *h* and *k* values of your function?
2. What is the domain? What value is not included in the domain? Why?
3. How would you describe the behavior of the function near x = 0, the vertical asymptote?
4. How would you describe the end behavior?
5. Enter the function  into Y1 on your calculator. Record the values for *a, h* and *k* below.

a. a= \_\_\_\_\_\_, h= \_\_\_\_\_\_, k = \_\_\_\_\_\_



b. How did the graph of the function shift? Describe the transformation.

c. What is the equation of the VA?\_\_\_\_\_\_\_ What is the equation of the HA? \_\_\_\_\_\_\_\_

d. How are the VA and the HA related to the *h* and *k* values of your function?

e. What is the domain? What value is not included in the domain? Why?

1. What is the relationship between the domain and the vertical asymptotes for the rational functions that you explored in the previous examples?
2. Summarize the effects that the parameters *h* and *k* have on the graph of .
3. Do you think that the graph of a rational function can ever touch or cross a VA? Explain.
4. Do you think that the graph of a rational function can ever touch or cross a HA? Explain.

13b.What is the relationship between the end behavior and the horizontal asymptote?

1. Describe the transformations (shifts) for each of the equations below.
   1.  b.  c. 
2. Now, let’s look at how the “*a*” parameter affects the graph of the function.





* 1. Enter into Y1 and graph the function. How did the function change?
  2. Enter into Y1 and graph the function. How did the function change?
  3. Enter into Y1 and graph the function. How did the function change?
  4. Enter into Y1 and graph the function. How did the function change?
  5. Summarize how the parameter “*a*” affects the graph of the function.

1. Draw the VA and the HA onto the graphs below. Then write an equation for the rational function associated with each graph. (Assume a = 1)

 

* 1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ b. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 

c. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ d. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Try to define a horizontal asymptote so that we are just not saying it is a special horizontal line.
2. Try to define a vertical asymptote so that we are just not saying it is a special vertical line.