**Activity 2.2.3 ASA Congruence**

In this activity you will discover and prove our second theorem about congruent triangles.



1. Included sides. For each pair of angles in ∆XYZ, name the included side.

 Angles: $∠$*XYZ* and $∠$*YZX* Included Side: $ \\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$

 Angles: $∠$*XYZ* and $∠$*ZXY* Included Side: $ \\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$

 Angles: $∠$*ZXY* and $∠$*YZX* Included Side: $ \\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$

2. Experiment. Work with one other student. You will each draw two triangles using a ruler and protractor.

a. Agree upon the measure of two angles of the triangle and the included side.

 Our two angles measure \_\_\_\_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

 Our included side measures \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

b. Now draw your triangles. Cut one triangle out and place it on the other. What do you notice?

c. Formulate a conjecture: If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.



3. Proving the ASA Congruence Theorem. Study this proof and fill in the blanks.

Given ∆*ABC* and ∆*DEF* with

m$∠$*BAC* = m$∠$*EDF*,

$m∠$*ABC* = m$∠$*DEF*, and

*AB* = *DE*

Prove ∆*ABC*$ ≅ $∆*DEF*.

*Step 1*. If *A* and *D* do not coincide then translate ∆*DEF* by the vector from *D* to *A*.

Now *D’* is the same point as \_\_\_\_\_\_\_.



*Step 2*. If $\overbar{AB}$ and $\overbar{D'E'}$ do not coincide then rotate ∆$D’E’F’$ about point *A* through $∠$*E’AB.*

*Step 3.* $\overbar{D’’E’’}$ will now coincide with $\overbar{AB}$ since we were given that *AB* = \_\_\_\_\_\_\_.

*Step 4.* If *F’’* is on the opposite side of $\overleftrightarrow{AB}$ from *C*, reflect ∆*D’’E’’F’’* over $\overleftrightarrow{AB}$.



*D’’* is the same as which point? \_\_\_\_\_

*Step 5*. $∠$*E’’’D’’’F’’’* will now coincide with $∠$*BAC* since we were given m$∠$*BAC* = m$∠$\_\_\_\_\_\_, and $∠$*D’’’E’’’F’’’* will now coincide with $∠$*ABC* since we were given m$∠$*ABC* = m$∠$\_\_\_\_\_\_.

6. Therefore ray $\vec{D’’’F’’’}$ coincides with ray $\vec{AC}$ and ray $\vec{E’’’F’’’}$ coincides with ray $\vec{BC}$. These two rays intersect at point *C*. Is it possible for them to intersect at another point? Explain. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

If we agree that two lines can only intersect at one point, we can conclude that point *F’’’* coincides with point *C.*

*Step 7*. Since ∆*ABC* is the image of ∆*DEF* under an isometry, ∆*ABC*$ ≅ $∆*DEF*.

4. Now state the theorem you have just proved. We will call this the ASA Congruence Theorem.

If \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_,

then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

5. What properties of transformations were used in the proof?

6. How was this proof similar to the proof of the SAS Congruence Theorem? How was it different?