**Unit 9: Functions and Inverse Functions**

**(*Modified version of Algebra 2 Unit 1*)**

**(4 Weeks)**

**UNIT PLAN**

This Unit serves as both a review and an extension of functions from Algebra 1. Students will be familiar with functions, particularly linear functions, from Algebra 1. These subjects are reviewed in new contexts, such as linear programming and conic sections. The Unit then extends students’ understanding of functions by examining the important concepts of function growth, transformation of functions, creating new functions from old, and inverse functions. This Unit sets the groundwork for the rest of Algebra 2 by investigating functions in a general way which is then applied as needed later in the curriculum. As many of the ideas in this Unit review concepts introduced in Algebra 1, there is an emphasis throughout Unit 1 on some of the applications of these ideas to realistic contexts.

**Investigation 1** *in Unit 8 of the Algebra 2 curriculum has already been covered in Unit 8 of Integrated Mathematics. Therefore this Unit will begin with Investigation 2.*

**Investigation 2** is primarily a review of concepts learned in Algebra 1. Here there is more emphasis on using functions as models for realistic situations. For example, students will investigate the domain and range of a function both from the viewpoint of the set of possible inputs to and outputs from a function, but also as a set of reasonable values in realistic contexts. Students will translate between representations of functions as graphs, tables of values, equations, and verbal descriptions. Students will also review the conic sections they learned in Geometry from the viewpoint of whether or not they can be represented as functions.

In **Investigation 3**, students are asked to compare and contrast linear and nonlinear functions in terms of growth. They will learn that for linear functions, the first difference is constant; for quadratic functions, the second difference is constant; and for exponential functions, the ratio of successive terms is constant. Students will also investigate the absolute value function as well as piecewise and step functions. Again, the emphasis is on the contexts in which these types of functions would arise. Finally, students will look at even and odd symmetry in functions, anticipating the polynomial and trigonometric functions they will study later in Algebra 2.

**Investigation 4** asks students to construct new functions from old functions. They will distinguish between “inside” changes, those applied to the independent variable; and “outside” changes, those applied to the dependent variable. Students will apply their understanding of inside and outside changes to functions to investigate the effects of the transformations f(x) + k, f(x + k), and kf(x) for a constant k. Students will also learn how to decompose functions so that they might think of complex functions as a “parent” function with one or more transformations applied to it. Finally, they will investigate transformations and the algebra of functions in the context of climate change and greenhouse gas emissions.

**Investigation 5** is intended for STEM-intending students. The Investigation focuses on function composition. Students will study the composition of functions as well as how to decompose a function into two simpler functions. They will determine the domain and range of the composite function, including a consideration of the case where the inner function has a restricted domain. These ideas are applied to a range of realistic contexts where the ideas of function composition arise naturally.

**Investigation 6** explores the concept of inverse functions. The concept of an inverse is presented informally at first, that an inverse function “undoes” what a function “does.” A function can only be “undone” in one way for the inverse to also be a function; that is, only 1-1 functions will have an inverse function. As students become comfortable with “doing” and “undoing,” we shift to “reversing” the roles of the independent and dependent variables, which leads to being able to find a formula for an inverse function. Doing so will extend to the case where a function must have its domain restricted in order for the inverse to be a function. In addition, STEM-intending students who have worked through Investigation 5 will determine whether a pair of functions are inverses by composing the functions. Students will then apply their knowledge of inverse functions to a range of realistic examples using inverses.

A particular example of inverse functions that are widely used are the root functions, especially $\sqrt{x}$ and $\sqrt[3]{x}$. In **Investigation 7**, students will study these functions using their knowledge of them as inverses of the functions x2 and x3 respectively. The ideas of root functions will be applied to the field of music. The standard way to tune music is to define the frequency of A above middle C as 440 Hz. Since one octave represents a doubling of the Hz, the A one octave higher is 880 Hz. There are 12 half-steps between these two notes, which means that every half-step between the two can be reached by a factor of $2^{\frac{1}{12}}$. This represents a very natural context for 12th roots.

**Essential questions**

* What is a function?
* What are the different ways in which functions may be represented?
* How do we find an optimal solution to a linear programming problem?
* What is the meaning of the domain and range of a function?
* What is a family of functions?
* How do different families of functions grow?
* What is the effect of a transformation of an independent variable? What is the effect of a transformation of a dependent variable?
* (+) What does it mean to compose two functions?
* What is the inverse of a function?
* How can functions be used to model real world situations, make predictions, and solve problems?

**Enduring Understandings**

* A function is a special kind of relation where each member of the domain of the function is associated with exactly one member of the range of the function.
* We can study a very wide range of phenomena by starting with a very small group of “parent” functions and applying transformations to those functions.

**Unit Understandings**

* Functions are a mathematical way to describe relationships between two quantities that vary.
* An essential characteristic of families of functions is the way in which they grow.
* Transformations of functions of the forms f(x) + k and f(x + k) have the effect of shifting functions up or down (in the case of f(x) + k), or left or right (in the case of f(x + k)).
* Transformations of functions of the forms kf(x) and f(kx) have the effect of stretching or compressing functions horizontally (in the case of f(kx)) or vertically (in the case of kf(x)).
* When it exists, an inverse of a function “undoes” what a function “does.” If we start with a value of the domain of a function, apply the function, then apply the inverse function, we should obtain the original value.
* Graphically solving a linear system of inequalities in two variables can be used to assist in finding the solution of a Linear Programming problem.
* A graphical solution of Linear Programming can be used to identify optimal solutions to practical problems in two linear variables.

**Unit Contents**

Investigation 2: Relations and Functions (2 days)

Investigation 3: Types of Functions (4 days)

Investigation 4: Building New Functions From Old (5 days)

Investigation 5 (+): Composition of Functions (3 days)

Investigation 6: Inverse Functions (3 - 4 days)

Investigation 7: Root Functions (2 days)

Mid-Unit Test (1 day)\*

Performance task (1 day)

Review of unit (1 day)

End of unit assessment (1 day)

\*See Mid-Unit Answer Key for teacher notes regarding problems to select

**Common Core Practice Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Common Core State Standards**

A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.D.11 Explain why the x-coordinate of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include where f(x) and/or g(x) are linear, rational, absolute value, exponential and logarithmic functions.

A.REI.D.12 Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality) and graph the solution set of a system of linear inequalities as the intersection of the corresponding half-planes.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then *f*(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = *f(*x).

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the* *function h(n) gives the number of person-hours it* *takes to assemble n engines in a factory, then the* *positive integers would be an appropriate* *domain for the function*.

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.BF.1b Combine standard function types using arithmetic operations.

F.BF.1c (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.3 Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.4 Find inverse functions.

F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2(x3) for x > 0 or f(x) = (x + 1)/(x – 1) for x ≠ 1 (x not equal to 1).

F.BF.4b (+) Verify by composition that one function is the inverse of another.

F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the graph has an inverse.

**Assessment strategies**

**Performance Task**

Students start with the outline of a corporate logo and reproduce it using functions.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* Unit 1 end of unit assessment

**Vocabulary**

Absolute value function

Algorithm

Angle of a generator (for conic sections)

Area of a circle

Axis of symmetry

Boundary line

Circle

Circumference

Composition of functions

Constraint

Dependent variable

Depreciation

Directrix

Domain

Ellipse

Equivalent equation and inequality

Even function

Exponential function

Family of functions

Feasible region

Floor function

Foci of an ellipse, hyperbola, or parabola

Function

Function notation

Greatest integer function

Half-plane

Horizontal and vertical axes

Horizontal line test

Horizontal shift/translation

Hyperbola

Independent variable

Inequality

Input and output of a function

Inside change

Inverse function

Isometry

Line symmetry

Linear growth

Major axis/minor axis of an ellipse

Maximum/minimum

Natural domain

Nonlinear growth

Objective function

Odd function

One-to-one (1-1) function

Optimization

Ordered pair

Origin

Outside change

Parabola

Parameter

Parent function

Piecewise function

Quadratic function

Quadrivium

Radius of a circle

Range

Reflection

Relation

Restricted domain

Root function

Rotational symmetry

Shading

Solution of a system of linear equations

Solution of a system of linear inequalities

Standard form (of an equation for conic sections)

Slope

Step function

Symmetric with respect to the origin

Symmetric with respect to the y-axis

System of equations

System of inequalities

Test point

Transformation

Vertex

Vertical stretch

Vertical shift/translation