**Unit 6: Transformations
(*Modified version of Geometry Unit 1*)**

**(4 Weeks)**

**UNIT PLAN**

This unit builds on the background students bring to this course. Ideally students should have had considerable exposure to geometry in the Grade 8 Core Curriculum. In practice we realize that in the mid-teens of this century this will not yet be the case for most students. However, we do expect students to be familiar with the names of geometric figures, methods for computing area and perimeter, and some intuitive understanding of geometric transformations. Furthermore from their Algebra 1 course they will be familiar with the coordinate plane, equations of lines and the meaning of slope. Therefore, in this informal introduction to geometry we will focus on coordinate geometry and the representation of transformations in the coordinate plane. As students discover the properties of transformations they will establish the basis for a series of postulates that will become the foundation of the logical structure of geometry beginning in Geometry Unit 2 (*Integrated Mathematics Unit 12*).

In this unit students will also be introduced to dynamic geometry software, such as Geogebra or Geometer’s Sketchpad. The activities that use Geogebra in this unit introduce students to various features of the program through explicit “how to” instructions. Once students become familiar with the software, they will no longer need detailed guidance as they use this tool to investigate properties of geometric figures in subsequent units. Some activities that are initially written for Geogebra will also have alternate versions for students using Geometer’s Sketchpad. In some cases there will also be a “low tech” version based on hands-on materials or explorations in the coordinate plane.

***Investigation 1 The Pythagorean Theorem and the Distance Formula*** *in Unit 1 of the Geometry curriculum has already been covered in Unit 5 of Integrated Mathematics. Therefore this Unit will begin with Investigation 2.*

**Investigation 2 Vectors and Translations** introduces the first transformation studied in this unit. Transformations are defined in terms of vectors, which can be indicated by the pair of motions [run, rise]. This concept is closely related to slope (from Algebra 1) and to the distance formula (introduced in the previous investigation).

The next transformation is introduced in **Investigation 3 Angles and Rotations.** Rotations are defined by a center point and a directed angle measure. Whereas students’ previous conception of angle may have been static (i.e. the angle as the union of two rays with a common endpoint), this approach linking angles to rotations is dynamic. Because Geogebra always measures angles in a counter-clockwise direction, the concept of directed angle measure is reinforced.

**Investigation 4** **Reflections** introduces the last of the three most common isometries. This investigation builds on students’ intuitive understanding of mirror images.

Composition of functions is an important concept in the second-year algebra course. This is foreshadowed here in **Investigation 5 Composition of Transformations.** Students discover what happens when one transformation follows another. They are also introduced to the fourth isometry—glide reflections, which are created by the composition of a translation and reflection over a line parallel to the translation vector.

**Investigation 6 Symmetry** provides an opportunity to apply what has already been learned about transformations to figures with lines of symmetry or rotational symmetry. This concept will be visited again in Unit 3 when students study regular polygons.

The unit culminates in **Investigation 7 Isometries.** The concept of isometry (also known as a rigid motion) is introduced by looking briefly at some transformations that are not isometries: dilations (to be developed more in Unit 4), stretches, and shears. Isometries are then defined as those transformations that preserve both distances and angle measures. Translations, rotations, reflections, and glide reflections are all isosmetries. The properties of all isometries and of specific isometries are codified into a series of postulates that will be used in Unit 2 to establish the properties of congruent figures.

The Performance Task for this unit will involve the application of transformations to a work of art.

**Essential Questions**

* How is coordinate geometry used to find distances and determine parallel and perpendicular lines?
* What are transformations?
* What properties are preserved by various transformations?
* What properties are common to all isometries?

**Enduring Understandings**

Geometric transformations may be explored through the application of concrete materials, coordinate geometry, and the use of dynamic geometric software.

**Unit Contents**

Investigation 2 Vectors and Translations (4 days)

Investigation 3 Angles and Rotations (2 days)

Investigation 4 Reflections (1 day)

Investigation 5 Composition of Transformations (3 days)

Investigation 6 Symmetry (2 days)
Investigation 7 Isometries (2 days)
Performance Task (1-2 days)

Review (1 day)

End of Unit Test (1 day) (Note: There is no mid unit test.)

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. Make sense of problems and persevere in solving them.

2. **Reason abstractly and quantitatively.**

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. **Use appropriate tools strategically.**

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Content Standards Overview**

* Understand congruence in terms of rigid motions

**Content Standards**

G-CO.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

8.G.6. Explain a proof of the Pythagorean Theorem and its converse.

8.G.7. Apply the Pythagorean Theorem to determine the unknown side lengths in right triangles in real-world mathematical problems in two and three dimensions.

8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

**Postulates and Theorems Introduced in This Unit**

**Pythagorean Theorem:** In any right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. (Informal proof) [Investigation 1]

**Distance Formula:** If the coordinates of two points in a plane are (*x*1, *y*1) and (*x*2, *y*2), then the distance between the two points is equal to $\sqrt{(x\_{2}-x\_{1})^{2}+(y\_{2}-y\_{1})^{2}}$. (Coordinate proof) [Investigation 1]

**Line Separation Postulate:** Every point on a line divides the line into two rays with only their endpoint in common. [Investigation 2]

**Parallel Line Slope Theorem:** If two lines in the coordinate plane have the same slope, then they are parallel. (From Algebra 1) [Investigation 2]

**Midpoint Formula:** If the coordinates of the endpoints of a segment in a plane are (*x*1, *y*1) and
(*x*2, *y*2), then the midpoint of the segment has coordinates $\left(\frac{x\_{1}+x\_{2}}{2},\frac{y\_{1}+y\_{2}}{2}\right)$. (Coordinate proof)

**Perpendicular Lines Slope Theorem:** If the slopes of two lines are opposite reciprocals, then the lines are perpendicular. (from Algebra 1) [Investigation 3]

**Plane Separation Postulate:** If a line *l* lies in a plane then every point in the plane not on *l* lies on one side or the other side of *l*. If *B* lies on the opposite side of *l* from *A,* then the line segment $\overbar{AB}$ intersects line *l* in one point. [Investigation 4]

**Isometry Postulate** (applies to Translations, Rotations, Reflections and Glide Reflections): All isometries preserve distance and angle measure. Pairs of parallel lines are mapped onto parallel lines. Midpoints of segments are mapped onto midpoints.

**Translation Postulate**: Under a translation by the vector from *X* to *Y*, a line parallel to $\overleftrightarrow{XY}$ is mapped onto itself. A line that is not parallel to $\overleftrightarrow{XY}$ is mapped onto another line that is parallel to itself.

**Rotation Postulate**: Under a rotation about a point *P*, the point *P* is mapped onto itself. A line through point *P* is mapped onto another line through *P*. If the angle of rotation is 180° a line through *P* is mapped onto itself.

**Reflection Postulate**: Under a reflection about line *l* every point on *l* is mapped onto itself. A line that is parallel to *l* is mapped onto another line that is parallel to *l*.

**Vocabulary**

altitude

angle

base

clockwise

composition (of transformations)
counterclockwise

dilation

distance

distance formula

endpoint

glide reflection
horizontal stretch

hypotenuse

image

isometry

isosceles trapezoid
isosceles triangle

leg

line

line of symmetry

mapping rule

midpoint

mirror line or reflection line
opposite vectors

parallel (lines)

parallel vectors
perimeter

perpendicular

point

polygon

pre-image

ray

reflection

right triangle

rise

rotation

rotational symmetry

run

segment

shear

side (of angle)
side (of polygon)

symmetry

transformation

translation
triangle

vector
vertical stretch

**Assessment Strategies**

**Performance Task**

Students will create a work of art using what they have learned about transformations.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* End of Unit test