**Unit 8: Investigation 3 (3 Days)**

**Further Investigation of Tessellations**

**Common Core State Standards (extended)**

* G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
* G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

**Overview**

Students will explore semi-regular tilings and other tilings. Their work reviews finding each angle in a regular polygon, reasoning about fitting regular polygons, and searching systematically for sums of 360°. They will examine conditions that allow a pentagon to tile the plane. Extensions involve demi-regular tilings, Escher drawings, and Penrose tiles.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Explain how to find each angle in a regular polygon.
* Distinguish regular, semi-regular, demi-regular, and other tilings.
* Use Schläfli symbols to describe tiles.
* Reason about why some tilings are impossible.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip** **8.3** asks students to symbolize two different vertices in a demi-regular tiling.
* **Journal Entry** asks student to explain why some sets of figures that can fill the space around one vertex are not able to create a semi-regular tessellation.

**Launch Notes**

This is an extension of Unit 3 Investigation 7. If students have not completed that investigation they should do so before proceeding. If they have you may want to review these important concepts:

A tessellation (or tiling) is a repeating pattern of polygons that if extended fills the entire plane. Here are the properties of tessellations established in Activity 3.7.1.

1. It is made of congruent figures.
2. Only one figure (or a small number of figures) are used.
3. There are no gaps between the figures.
4. If three or more figures meet, they do so only at a vertex. (When this condition is met we have an **edge-to-edge** tiling. We will see examples in Activity 8.3.2 where this condition is not met).

(5) The tessellation continues indefinitely and fills the entire plane.

Also recall that a regular tessellation consists only of one type of regular polygon. Activity 3.7.2 established that the only three figures that form regular tessellations are the equilateral triangle, the square, and the regular hexagon.

In Activities 3.7.3 and 3.74 it was established that any triangle or any quadrilateral will tile the plane. However, pentagons do not have this property. Students will have come to this understanding in Activity 3.7.2 when they attempted to tile the plane with regular pentagons. Activity 8.3.2 gives examples of tilings with equilateral (but not regular) pentagons.

Schläfli symbols were introduced in Activity 6.1.3 to describe the configuration of regular polygons around a vertex. They are used here as well to describe vertices in a tessellation. Thus 3.3.3.4.4 indicates three equilateral triangles and two squares share a vertex. The order the figures are named is important; that is 3.3.4.3.4 is not the same as 3.3.3.4.4.

Once these concepts have been reviewed or explored for the first time, we are ready to move on.

**Differentiated Instructions (for learners needing more help):**

A brief review of Unit 3 Investigation 7 may not be sufficient to prepare some students for this investigation. Consider reassigning some of the earlier activities, especially Activity 3.7.2.

**Teaching Strategies**

**Activity 8.3.1 Possible Semi-regular Tilings** has students reason about what must be true for different numbers or sets of tiles to ‘fill’ a point (total 360°). Students also search for sums of 3, 4, or 5 regular polygons that can fill a point. Finally they use tiles to figure out which of those possibilities really work to fill the plane. Students who have studied Archimedean solids will find the arguments eliminating certain vertex configurations are similar to the ones used in Activity 8.7.1.

For this activity each team of students will need a kit of cut outs of regular polygonal tiles with the same sized edges. A template for such polygons is provided in the teacher notes.

Or use NCTM software at <http://illuminations.nctm.org/Activity.aspx?id=3533>

**Journal Prompt:** Explain why not all sets of regular polygons that can fit around a point will necessarily tile the plane. Look for students to show by example that when a polygon with an odd number of sides is matched with two different types of polygons, you are unable to continue the pattern at every vertex.

In **Activity 8.3.2 Pentagonal Tiling** students explore which types of pentagons can tile the plane. They start with the “house” pentagon formed by an equilateral triangle and a square and look at ways it can be modified and still tile the plane. This is a good place for students to use dynamic software such as GeoGebra in their exploration. Students are then shown some tilings that do not meet the edge-to-edge restriction and they are then referred to web sites whey they can learn about the various recipes that have been found for creating pentagonal tilings. Point out that this in an area in which research is ongoing and that an “amateur” mathematician, Marjorie Rice, made some of the recent discoveries. (See https://en.wikipedia.org/wiki/Marjorie\_Rice)

**Activity 8.3.3 Extensions for Tilings** provides suggested projects related to tilings for further exploration. These include demi-regular tilings, Escher type tilings, and Penrose tilings.

**Exit Slip 8.3** may be given following Activity 8.3.3.

**Closure Notes**

Close the investigation by having students who have worked on Activity 8.3.3 present their reports.

**Vocabulary**

demi-regular tiling

edge-to-edge tiling  
Escher-like tiling

Penrose tiling

regular polygon

regular tiling

Schläfli symbol

semi-regular tiling

tessellation

**Resources and Materials**

Activity 8.3.1 Possible Semi-regular Tilings

Activity 8.3.2 Pentagonal Tiling

Activity 8.3.3 Extensions for Tilings

Exit Slip 8.3

Template for polygonal tiles (for Activity 8.3.1) or use NCTM software at <http://illuminations.nctm.org/Activity.aspx?id=3533>

Newman, Rochelle & Martha Boles (1992). *The Surface Plane*. Bradford, MA: Pythagorean Press.

Elliott, H. A., James R. MacLean, & Janet M. Jordan. (1968). *Geometry in the Classroom: New Concepts and Methods*. Toronto: Holt, Reinhart and Winston of Canada. (See pp. 150-152 for ways to transform triangles to tile the plane.)

<http://paulscottinfo.ipage.com/polyhedra/tessellations/semiregulartess.html>

<http://paulscottinfo.ipage.com/polyhedra/tessellations/semiregular_construct.html>

<http://mathforum.org/dynamic/one-corona/>

<http://www.mcescher.com>

<http://illuminations.nctm.org/Activity.aspx?id=3533>

<http://curiosamathematica.tumblr.com/post/76886217509/penrose-tilings-are-non-periodic-tilings>

<http://www.mathpuzzle.com/tilepent.html>

<https://en.wikipedia.org/wiki/Pentagonal_tiling#CITEREFBagina2011>

<https://sites.google.com/site/intriguingtessellations/home>