**Unit 8: Investigation 1 (2–3 Days)**

**Frieze Patterns**

**Common Core State Standards (extended)**

* G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
* G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

**Overview**

In spite of the fact that there are infinitely many art forms for repetitive border designs, there are only seven different ways by which such designs can be generated from an asymmetrical figure by performing isometric transformations. In this investigation we try to deepen students’ understanding of symmetries and transformations and their value in art and culture.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Given a frieze pattern, students will identify transformations that are present.
* Given a frieze pattern, they will use the Conway or IUC system to classify it.
* Given one of the seven basic patterns, students will create a frieze that matches the design.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit slip** **8.1** presents students with four frieze patterns and asks them to classify them.
* **Journal entry** asks students to describe the transformations that are used to create a frieze pattern.

**Launch Notes**

Introduce students to the concept of a border pattern (or frieze pattern) by showing examples from various forms of art. A short video (less than two minutes) that does this is found at

<https://www.youtube.com/watch?v=AjWjHtjQW_E>.

**Teaching Strategies**

**Activity 8.1.1 Borders in Cultures** intends to engage students in ‘seeing’ the rotations (half-turns), horizontal reflections (over a symmetry line parallel to edges of border), vertical reflections (over lines perpendicular to edges of border), and glide reflections.

* Begin by inviting students to share what they like about the designs. Which are their favorites? Where do they find border designs around them? Which might they use and how?
* Ask students how the patterns repeat. They should notice that a portion of the pattern is translated horizontally over and over again. The smallest horizontal vector that maps the pattern onto itself may be called the “translation vector” of the frieze pattern.
* Encourage students to use miras to find reflection lines and tracing paper to find centers of rotations (by doing the half turn, checking to see if the shape overlays on itself). Have students show their thinking at the document camera or by having others cluster around their desk.
* Encourage students to try to generate language or ways of explaining where they see ‘flips,’ ‘half-turns,’ or ‘walking steps’ (glide reflection.) In later activities, strive to attach student-generated language to the other ways of explaining (mathematical language, IUC notation, and Conway’s dance steps.)
* These explorations should help students realize that a frieze pattern can have at most one horizontal reflection line. However, if there is one vertical reflection line there are an infinite number of them, the distance between them being half the length of the translation vector.

**Activity 8.1.2 Creating Seven Types of Border Designs** helps students develop a more kinesthetic sense of how the designs are generated and to continue to connect actions and language to simplified versions of the designs.

* At first students use machine tape and templates to extend patterns that have been begun. Students may use cardboard cut out congruent scalene triangles or templates in which they translate, reflect, or rotate the congruent shapes.
* We also introduce Conway’s dance terms to characterize the seven designs. You may want to take students in the hallway and have them move as each name indicates. Or, in groups of 6 or 12 they could stand in ways that show how the feet are moving along. As they do, again, ask them why Conway’s terms make sense and how the terms can remind them what they are looking for. (E.g., *spinning* means there is a half-turn.)

**Differentiated Instruction (For Learners Needing More Help)**

Acting out Conway’s seven dance steps is helpful for students who are kinesthetic learners to internalize the seven patterns.

**Activity 8.1.3 Classifying and Analyzing Border Designs** supports students synthesizing what they have observed to arrive at a systematic method for analyzing border designs.

* Begin by having students study the partially started chart synthesizing their work to date. What kind of information goes in each column? Which cells can they fill from just the clues given, without looking back at previous sheets?
* As students complete the chart, listen for debates among the students and have one of those re-enacted for the entire class. Let others clarify and re-voice final decisions.
* Have students return to the art examples in Activity 1 and formalize which type of design each one is.
* Have students study the flowchart and the IUC notation and reflect on how these tools may help them to classify the designs.

**Exit Slip 8.1** may be given following **Activity 8.1.3.**

**Activity 8.1.4** **Generating the Seven Patterns** helps reveal why there are precisely seven border designs.

**Journal Entry.** Explain to a friend how geometric transformations are used to create border patterns. Look for students to explain that all border patters have translations and that some have reflections, rotations, and/or glide reflections.

**Differentiated Instruction (Enrichment)**

Students who are interested in a formal proof of that there are exactly seven different types of frieze pattern should be encouraged to read the article by Doris Schattschneider referenced in **Activity 8.1.5.**

If time permits, or for individual exploration, you may assign **Activity 8.1.5.**

**Activity 8.1.5** **More Explorations with Frieze Patterns** invites students to draw upon their artistic talents (musical, graphic arts, dance, computer design, gaming) and apply them to some creative expression that embodies their understanding of these seven designs. With enough variety of different student pairs or small groups, the class or a number of classes might collaboratively create a “Frieze Design Extravaganza.” Ideally, the student products or productions could help family and community members learn the mathematics of frieze designs.

**General Rubric for Projects based on Activity 8.1.5 with suggested weighting**

1. A **plan** for the project was completed on time and with enough detail for the teacher or peers to give useful feedback. The final project reflects integration of some of the feedback. (20%)
2. **Mathematics** is accurate, coherent, and complete. Students make clear their firm knowledge of the topic, their understanding of core mathematical relationships. (30%)
3. **Visuals** are done with precision, well organized. (10%)
4. Work shows some **creativity** in conceptualization or presentation. (10%)
5. The project overall has **coherence**. (10%)
6. Written and oral elements use standard **English**. (10%)
7. All **partners** contributed and supported one another and explained theirs and others contributions in a self-evaluation. (10%)

**Group Activity**

Have students work in groups to create projects based on the ideas suggested in **Activity 8.5.1.**

**Closure Notes**

Have students summarize what they have learned about frieze patterns.

**Vocabulary**

Conway classification system

flowchart for classifying patterns

frieze

IUC (International Union of Crystalography) system of classification

**Resources and Materials**

Activity 8.1.1 Borders in Cultures

Activity 8.1.2 Creating Seven Types of Border Designs

Activity 8.1.3 Classifying and Analyzing Border Designs

Activity 8.1.4 Generating the Seven Patterns

Activity 8.1.5 More Explorations with Frieze Patterns

Exit Slip 8.1

Miras (used in Activities 8.1.1, 8.1.2)

Tracing paper (used in Activity 8.1.1)

Adding Machine Tape (used in Activity 8.1.2)

Video: <https://www.youtube.com/watch?v=AjWjHtjQW_E>.

Print media:

Gerdes, Paulus. *Geometry from Africa: Mathematical and Educational Explorations*. Washington, D.C.: Mathematical Association of America, 1999.

Harigittai, István and Magdolna Hargittai. *Symmetry: A Unifying Concept.* Bolinas, CA: Shelter Publications, 1994.

Martin, George E. Transformation Geometry, An introduction to symmetry, Undergraduate Texts in Mathematics, Springer Verlag, 1982.

Schattschneider , Doris. “What Would Sherlock Do?” in *Understanding Geometry for a Changing World*, Craine and Rubenstein, ed. Reston, VA: National Council of Teachers of Mathemaitcs, 2009.