**Activity 8.5.1 Solving A Recycling Matrix Equation**

Here is a quick review of solving a linear system with two variables with a matrix equation:

Two basketball teams are going into the third period with Team A leading Team B 70 to 64. Team A has made as many three point shots as team B has made two point shots. Team B has made two fewer three point shots than Team A has made two point shots. They both have made ten free throws. How many of each shot did each team make?

Assign variables:

Let *x* be the number of three point shots made by team A

Let *y* be the number of two point shots made by team A.

1. Write the point equation for both teams:
2. Write the matrix equation:
3. Use the formula for the inverse matrix: to find the inverse matrix:
4. Multiply the inverse matrix byto find the solution:

Verify that your results match these:

Team A scored 12 threes and 12 twos for a total score of 36 + 24 + 10 = 70

Team B scored 10 threes and 12 twos for a total score of 30 + 24 + 10 = 64

The same procedure can be used to solve equations involving three variables. For example, your school’s reduce/reuse/recycle initiative not only includes textile collection, but encouraging families to improve recycling efforts with bottles, cans, and newsprint. This naturally leads to assembling and analyzing data which is often linear and often involves more than two variables. For example:

The equation for the value in dollars of the collected items is: (1)

The equation for the weight in pounds of the collected items is: (2)

The equation for the reduction in pounds of CO2 emissions is: (3)

1a. Explain the meaning of the term 0.05*x* in equation (1). \_\_\_\_\_\_

1b. Explain the meaning of the term 0.045*y* in equation (2). \_\_\_\_\_\_\_\_\_\_\_

1c. Explain the meaning of the term 2.5*z* in equation (3).\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Suppose we wished to know what combination of aluminum cans, plastic bottles and newsprint collected would result in $1000 gained, 1000 pounds collected, and a 5000 pound reduction in carbon dioxide emissions into the atmosphere? This can be solved using the exact same strategy we used with two variables.

Write the matrix equation using the coefficients for the three equations:

The next step is to find the inverse of the matrix. We have not yet learned how find the inverse of a 3×3 matrix by hand, so find the inverse with your graphing calculator.

Check to see that your result matches this matrix:

Solve the system by multiplying by the inverse matrix using your graphing calculator:

1. So in order to have $1000 gained, 1000 pounds collected, and a 5000 pound reduction in CO2, we must collect how many bottles, cans and pounds of newspaper?

As in this example, we frequently use technology to find the determinant and the inverse of a 3×3 matrix. There is the method for finding them by hand.

The method uses minors, cofactors and determinants so we first need to learn how to find minors, cofactors and determinants. We use as an example the 3×3 matrix:

A minor for an element aij in a square matrix B is a real number that is the determinant of a smaller matrix within a 3×3 matrix formed by crossing out row *i* and column *j*. For example, to find the minor of a11, cross out the first row and the first column and find the determinant of the matrix formed by remaining entries. The determinant of a matrix is written with straight bars instead of brackets.

The minor for element is the determinant of matrix which is written as:

The minor for element is

The minor for element is

There are nine minors for a 3×3 matrix. Why? \_\_\_\_\_\_\_\_\_.

Here are a few more examples:

The minor for element is

The minor for element is

1. Find the remaining four minors for matrix *A.*

Minors are used to find determinants and also to find the inverse of a matrix. First let’s see how we use them to find determinants. First a minor must be multiplied by a factor or 1 or -1 to determine a cofactor. A cofactor can be defined as Cij = (-1)i+j(Mij) where Mij is the corresponding minor.

We find the determinant of a square matrix by using the cofactors ***across any row or down any column***. For example, to find the determinant of matrix *A* by going across the first row, we will need the cofactors for the three elements in the first row and will then find their sum.

This is how it works:

To find the determinant of which we write as: by cofactor expansion across the first row:

(1)

To find the determinant working down the third column:

Notice we got same result as expected. There are four other ways to compute the determinant of this 3×3 matrix using cofactor expansion.

1. Compute the determinant using the second row.
2. Compute using the third row.
3. Compute using the first column.
4. Compute using the second column.
5. Did they all match?

The determinant of this matrix is 0. It can be mathematically proven that only matrices with nonzero determinants have an inverse. Recall that we did this for 2 by 2 matrices in investigation four. This matrix is singular and does not have an inverse.