**Activity 8.2.3 Characteristics of a Hypercube**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Figure | Dimensions | # of vertices | # of edges | # of faces | # of spaces |
| point | 0 | 1 | --- | --- | --- |
| segment | 1 | 2 | 1 | --- | --- |
| square | 2 | 4 | 4 | 1 | --- |
| cube | 3 | 8 | 12 | 6 | 1 |

1. Examine the table above.  Check the table you made in Activity 8.2.2 to see if you are in agreement.  Look for patterns and describe any patterns you observe.
2. Suppose we extend the table to four dimensions.  We add a row titled “hypercube” and a column called # of hyperspaces.  See if you can predict:

a. The number of vertices.  Look for a pattern in the column: 1, 2, 4, 8, \_\_\_\_.

b. The number of spaces.  Look for a pattern on the diagonal 0, 2, 4, 6, \_\_\_\_.

c. Notice that starting with number of edges, each number is equal to the sum of twice the number above it and the number above and to the left.  Use this to predict the number of edges and faces in a hypercube.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Figure | Dimensions | # of vertices | # of edges | # of faces | # of spaces | # of hyper-spaces |
| point | 0 | 1 | --- | --- | --- | --- |
| segment | 1 | 2 | 1 | --- | --- | --- |
| square | 2 | 4 | 4 | 1 | --- | --- |
| cube | 3 | 8 | 12 | 6 | 1 | --- |
| hypercube |  |  |  |  |  |  |

3.   Imagine, now, moving a cube into the fourth dimension.  Each feature duplicates itself and leaves a trail in the next dimension.  Fill in the chart below:

8 vertices result in \_\_\_\_ vertices + \_\_\_\_\_ edges

12 edges result in                           \_\_\_\_\_edges and \_\_\_\_\_\_ faces

6 faces result in                                                        \_\_\_\_\_faces and \_\_\_\_spaces

1  space results in                                                                                 \_\_\_spaces  and 1 hyperspace

Total result:  \_\_\_\_ vertices \_\_\_\_\_edges \_\_\_\_\_faces \_\_\_\_\_\_ spaces \_\_\_\_\_ hyperspace.

4. Check your results in question 3 with the table in question 2.

5. Recall Schlegel diagrams from Unit 6.  Here is a two dimensional Schlegel diagram of a four- dimensional hypercube. The inside cube appears to be smaller than the outer cube but in four dimensions they would be the same size.  Use the diagram to show each of the features of the hypercube found in questions 2 and 3 above.

