**Activity 8.4.4 Transforming Vectors with Matrices**

In the last activity, we multiplied matrices by vectors to solve equations using the inverse of a matrix. We know that when a matrix multiplies a vector, the result is vector because when we multiply an *m* × *n* matrix by an *n* × 1 vector the result is a *m* × 1 vector.

If *m* and *n* are equal then we are multiplying a square *m* × *m* matrix by an *m* × 1 vector and this gives a vector of the same dimension. So in that case, we transform one vector to another by multiplying by a matrix.

1. The simplest example is multiplying a vector by the identity matrix. What happens?

$$\left[\begin{matrix}1&0\\0&1\end{matrix}\right]\left[\begin{matrix}a\\b\end{matrix}\right]=\left[\begin{matrix}\\\end{matrix}\right]$$

1. Another simple example is $\left[\begin{matrix}2&0\\0&2\end{matrix}\right]\left[\begin{matrix}a\\b\end{matrix}\right]=\left[\begin{matrix}\\\end{matrix}\right]$
2. In this case, what happens to vector $\left[\begin{matrix}a\\b\end{matrix}\right]$ ?
3. Find the inverse of $\left[\begin{matrix}2&0\\0&2\end{matrix}\right]$
4. What happens when we multiply $\left[\begin{matrix}2&0\\0&2\end{matrix}\right]\left[\begin{matrix}a\\b\end{matrix}\right] by the inverse of \left[\begin{matrix}2&0\\0&2\end{matrix}\right]?$
5. So multiplying a vector by a matrix gives another vector. What do you think happens when we multiply the result by the inverse of the original matrix?
6. Lets check this out with matrix $\left[\begin{matrix}2&1\\1&2\end{matrix}\right]$.

What does this do to the original vector?

1. Find the inverse of matrix $\left[\begin{matrix}2&1\\1&2\end{matrix}\right]$
2. Check that multiplying by this inverse results in undoing the effect of multiplying vector $\left[\begin{matrix}a\\b\end{matrix}\right] $by $\left[\begin{matrix}2&1\\1&2\end{matrix}\right].$ Start with
3. Try this matrix:

$$\left[\begin{matrix}cos⁡(\frac{π}{4})&cos⁡(\frac{3π}{4})\\sin⁡(\frac{π}{4})&sin⁡(\frac{3π}{4})\end{matrix}\right]$$

What does this matrix do to the vector it multiplies?

Start with the vectors $\left[\begin{matrix}1\\0\end{matrix}\right]$ and $\left[\begin{matrix}0\\1\end{matrix}\right]$ and plot the location of the resultant vectors.

See if you can find a pattern and predict what multiplying by $\left[\begin{matrix}cos⁡(\frac{π}{4})&cos⁡(\frac{3π}{4})\\sin⁡(\frac{π}{4})&sin⁡(\frac{3π}{4})\end{matrix}\right]$ does to the original vector.



We have seen that multiplying a vector by a matrix transforms the vector to a new vector by rotation or stretching/compressing or a combination of these depending on the entries in the matrix. We will soon extend this to vectors in three-space.