**Activity 8.2.4 The Zero Vector and Subtracting Vectors**

In this activity we will examine the ways in which vectors are subtracted. Similar to vector addition, a vector’s representation impacts the manner in which we subtract vectors.

1. Let’s begin by working with arrows. Using a large sheet of graph paper and a vector, consider what vector $\vec{w}$ would have to be like in order to have the property that $\vec{v}+\vec{w}=\vec{v}$.
2. Let’s consider the vector addition shown below.



$$\vec{w}$$

$$\vec{v}$$

$$\vec{v}+\vec{w}$$

What vector $\vec{w}$ could have the property that $\vec{v}+\vec{w}=\vec{v}$. Look at the ordered-pair representations below to help.

$$\vec{v}=\left(v\_{1},v\_{2}\right)=\left[\begin{matrix}v\_{1}\\v\_{2}\end{matrix}\right] \vec{w}=\left(w\_{1},w\_{2}\right)=\left[\begin{matrix}w\_{1}\\w\_{2}\end{matrix}\right]$$

$$\vec{v}+\vec{w}=\left(v\_{1}+w\_{1},v\_{2}+w\_{2}\right)=\left[\begin{matrix}v\_{1}+w\_{1}\\v\_{2}+w\_{2}\end{matrix}\right]$$

The key to solving Questions 1 and 2 is the *zero vector*. The zero vector (in 2-D space) is the vector:

$$\vec{0}=\left(0,0\right)=\left[\begin{matrix}0\\0\end{matrix}\right]$$

So, for any vector $\vec{v}$,

$$\vec{v}+\vec{0}=\left(v\_{1}+0,v\_{2}+0\right)=\left[\begin{matrix}v\_{1}+0\\v\_{2}+0\end{matrix}\right]=\left(v\_{1},v\_{2}\right)=\left[\begin{matrix}v\_{1}\\v\_{2}\end{matrix}\right]=\vec{v}$$

1. Next, think about how the ordered pair notation can help you decide that for a vector $\vec{v}$ what vector $\vec{w}$ would have the property that $\vec{v}+\vec{w}=\vec{0}$?

$$\vec{v}+\vec{w}=\left(v\_{1}+w\_{1},v\_{2}+w\_{2}\right)=\left[\begin{matrix}v\_{1}+w\_{1}\\v\_{2}+w\_{2}\end{matrix}\right]=\vec{0}=\left(0,0\right)=\left[\begin{matrix}0\\0\end{matrix}\right]$$

In order for the sum $\vec{v}+\vec{w}$ to be 0, $v\_{1}+w\_{1}=0$ and $v\_{2}+w\_{2}=0$.

1. What would $w\_{1}$ need to be?
2. What would $w\_{2}$ need to be?
3. Look at the diagram below. Check to see if $\vec{v}+\vec{w}=\vec{0}$.



$$\vec{v}$$

$$\vec{w}$$

When $\vec{v}+\vec{w}=\vec{0}, we say that \vec{w}=\vec{-v}$ and we use this idea to subtract vectors. To subtract a vector $\vec{u} from a vector \vec{v}$, we can add $\vec{v}+\vec{-u}$. This is similar to the way that we can subtract scalar quantities. To subtract $3-\left(-5\right)$, we add the opposite of –5 to 3, and the resulting sum is 8.

The diagram shows $\vec{v}+\vec{u}$ and $\vec{v}-\vec{u}=\vec{v}+\vec{-u}$.



$$\vec{v}+\vec{u}$$

$$\vec{v}$$

$$\vec{v}-\vec{u}$$

$$\vec{u}$$

$$\vec{-u}$$

1. Determine the ordered pair notation for $\vec{v}, \vec{-u} and \vec{v}-\vec{u}$.
2. Verify that $\vec{v}-\vec{u}=\left(v\_{1}+\left(-u\_{1}\right),v\_{2}+\left(-u\_{2}\right)\right)=\left(v\_{1}-u\_{1},v\_{2}-u\_{2}\right)=\left[\begin{matrix}v\_{1}-u\_{1}\\v\_{2}-u\_{2}\end{matrix}\right]$
3. If $\vec{a}$ = (0, 3) and $\vec{c}$ = (5,0) then find $\vec{a}$ - $\vec{c}$ .
4. If $\vec{a}$ = (1, -2) and $\vec{c}$ = (4, -5) then find $\vec{a}$ - $\vec{c}$ .
5. $ \vec{r}=\left[\begin{matrix}7\\-9\end{matrix}\right], \vec{c}=\left[\begin{matrix}-4\\12\end{matrix}\right],\vec{v}= \vec{r}-\vec{c}=$