**Activity 8.1.5 Laws of Matrices**

We have performed a variety of operations on matrices and seen that they have many interesting properties. The focus of this activity is to explore how properties of real numbers enable similar behaviors for matrix operations but not in all instances due to the nature of the definitions of the matrix operations. Remember, for a property to indeed be a property of matrices, it must hold true for *all* matrices. As you saw earlier in this investigation, because of the nature of the definition of addition of matrices in terms of their real number elements, since addition for real numbers is commutative, addition of matrices is also commutative. So not only are we exploring properties of matrices but we are examining the nature of what it means for an operation to be commutative. If you elect to study additional mathematics you will encounter other operations on mathematical objects that may or may not be commutative.

Review of Real Number Properties that have Analogies for Matrices

* Addition of matrices is closed when adding two matrices of the same size. (If you add two matrices of the same size, the result will be a matrix of that size. If two matrices don’t have the same size, they can’t be added.)
* Multiplication of a matrix by a scalar gives a matrix.
* Addition of matrices is commutative. $A+B=B+A$
* Subtraction of matrices is not commutative.
* Multiplication of matrices is not commutative.
* Multiplication by a scalar is defined and multiplication by a scalar *k* is distributive: $k\left(A+B\right)=kA+kB.$
* Each matrix has an additive inverse—an opposite matrix.
* The zero matrix has the property that for all *A*, $A+0=0+A=A.$
* The identity *n* × *n* matrix *I* has the property that for all *A*, $A\*I=I\*A=A.$
* If a matrix *A* has an inverse, we call the inverse $A^{-1}$, and $A\*A^{-1}=A^{-1}\*A=I$—still to be addressed in investigations 3 and 4.

So far we have not looked into the existence associative laws for addition or multiplication, inverses under multiplication, and distributive properties for multiplication of matrices over addition.

In this activity we will first look at the distributive property for multiplication using specific 2 × 2 matrices. Then we will expand to “general” 2 × 2 matrices.

You will fill in the missing calculations.

 *A* = $\left[\begin{matrix}1&9\\5&6\end{matrix}\right]$ *B* = $\left[\begin{matrix}2&4\\11&12\end{matrix}\right]$ *C* = $\left[\begin{matrix}5&4\\8&7\end{matrix}\right]$

We first find find $A+B$.

$$A+B=\left[\begin{matrix}1+2&9+4\\&\end{matrix}\right]$$

Now we multiply $\left(A+B\right)C=\left[\begin{matrix}3&13\\&\end{matrix}\right]\*\left[\begin{matrix}5&4\\8&7\end{matrix}\right]=$

1. Fill in the missing parts of the product matrices below:

$$\left[\begin{matrix}\left(3\right)5+\left(13\right)8&\left(3\right)4+\left(13\right)7\\&\end{matrix}\right]$$

1. Simplify:

$$\left[\begin{matrix}119&103\\&\end{matrix}\right]$$

1. Find AC. Find BC:

AC =$ \left[\begin{matrix}&\\&\end{matrix}\right]=$

BC =$ \left[\begin{matrix}&\\&\end{matrix}\right]=$

1. $AC+BC$ =
2. Create 2 × 2 matrices *A*, *B*, and *C*.
3. Find $C\left(A+B\right)$.
4. Find $CA+CB$.
5. How do your results compare with those of other class members?

Now we will look at the distributive property for multiplication using general 2 × 2 matrices.

You will fill in the missing calculations.

 *A* = $\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$ *B* = $\left[\begin{matrix}e&f\\g&h\end{matrix}\right]$ *C* = $\left[\begin{matrix}k&l\\m&n\end{matrix}\right]$

We first find find $A+B$.

$$A+B=\left[\begin{matrix}a+e&b+f\\&\end{matrix}\right]$$

Now we multiply $\left(A+B\right)C=\left[\begin{matrix}a+e&b+f\\&\end{matrix}\right]\*\left[\begin{matrix}k&l\\m&n\end{matrix}\right]=$

1. Fill in the missing parts of the product matrices below:

$$\left[\begin{matrix}\left(a+e\right)k+\left(b+f\right)m&\left(a+e\right)l+\left(b+f\right)n\\&\end{matrix}\right]$$

1. Explain what we did in the next step:

$$\left[\begin{matrix}ak+ek+bm+fm&al+el+bn+fn\\&\end{matrix}\right]$$

1. Explain what we did in the next step:

$$\left[\begin{matrix}ak+bm+ek+fm&al+bn+el+fn\\&\end{matrix}\right]=$$

1. Explain what we did in the next step:

$$\left[\begin{matrix}ak+bm&al+bn\\&\end{matrix}\right]+\left[\begin{matrix}ek+fm&el+fn\\&\end{matrix}\right]=$$

$$AC+BC$$

1. What properties of real numbers were used in this proof?
2. Final result: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_