**Activity 6.1.4 Arc Length as Direct Variation**

Example: Find the length of an arc on a circle with radius 7 cm that is subtended by an angle that measures 3 radians. (Hint: imagine wrapping 3 radii around the outside of the circle as pictured below

Ans: There are 3 “radii” lengths wrapped along the arc, and each radius length is 7 cm,

so the arc length = (length of the radius)( radian measure)

 = (7cm) (3)

 arc length = 21 cm

1. Find the arc lengths subtended by the following central angles given in radian measure. Be sure your answer includes units of measure. As you answer these questions, try to figure out a formula that relates arc length to the radian measure of the central angle it subtends. Traditionally we use these variables:

‘s’ for arc length,

‘r’ for the length of the radius,

‘𝜽’ for the radian measure of the central angle subtended by the arc.

As you do these problems, envision how you would sketch the arc that is the correct number of radii wrapped around the circumference of the circle.

a. s = \_\_\_\_\_\_\_\_\_\_\_\_\_ if r =7 cm and 𝜽 = 1

b. s = \_\_\_\_\_\_\_\_\_\_\_\_\_ if r =7 cm and 𝜽 = 5

c. s = \_\_\_\_\_\_\_\_\_\_\_\_\_ if r =10 inches and 𝜽 = 1

d. s = \_\_\_\_\_\_\_\_\_\_\_\_\_ if r =10 inches and 𝜽 = 6

e. s = \_\_\_\_\_\_\_\_\_\_\_\_\_ if r =10 inches and 𝜽 = 2π

 (Note that this arc length is the circumference of the circle: C = 2πr)

f. The formula for arc length is: **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**You can use the formula to fill in the following blanks:**

g. s = \_\_\_\_\_\_\_\_\_\_\_\_\_ if r =1 meter and 𝜽 = 1

h. s = \_\_\_\_\_\_\_\_\_\_\_\_\_ if r =1 meter and 𝜽 = 5

i. s = \_\_\_\_\_\_\_\_\_\_\_\_\_ if r =1 meter and 𝜽 = 6π

j. s = \_\_\_\_\_\_\_\_\_\_\_\_\_ if r =8 meter and 𝜽 = $\frac{3π}{4}$

k. s = 12 if r = \_\_\_\_\_\_\_\_ meters and 𝜽 = 2

l. s = 24 if r =\_\_\_\_\_\_\_\_meters and 𝜽 = \_\_\_\_\_\_\_\_

m. If the radius of the circle = 1 unit, the arc length is equal to \_\_\_\_\_\_\_\_\_

n. Using the formula ‘s = r 𝜽’ for arc length, substitute 1 in for ‘r’ and simplify to obtain a special arc length formula for the unit circle. \_\_\_\_\_\_\_\_\_\_

2. You may have learned the arc length formula in geometry class. If you sketch concentric circles with different radii, you may see that circles are similar figures, and that therefore, the corresponding lengths are proportional.

a. Sketch two concentric circles,

one with radius 1 cm and the

other with radius 3 cm.

b. In your sketch, mark off an

arc subtended by a central angle

that measures one radian.

c. The length of the arc of the smaller

 circle is: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

d. The length of the arc of the larger

circle is: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

e. The arc length of the larger circle (radius 3 cm) is \_\_\_\_\_\_\_\_ times the arc length of the smaller circle (radius 1 cm.)

f. In algebra 1, you learned the slope intercept form of a line is y = mx + b and in unit 4 of this course you reviewed direct variation. If b = 0, then the equation y = mx + 0 can be written as y = kx, that is a formula for direct variation. The constant of proportionality is k. Suppose we keep the radius of the circle constant and vary the measure of the central angle. In your formula for arc length, ‘s = r 𝜽’, what letter could take the place of:

y? \_\_\_\_\_\_\_\_\_\_\_\_ k? \_\_\_\_\_\_\_\_\_\_\_\_\_\_ x? \_\_\_\_\_\_\_\_\_\_\_\_\_\_

g. The constant of proportionality in the arc length formula is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_of the circle.

We can say that the arc length “is proportional to” or “varies directly as” the central angle that subtends the arc, (the angle must be measured in radians), and the length of the radius is the constant of proportionality.