**Unit 7: Investigation 3 (3 Days)**

**Independent Events and the Multiplication Rule**

**Common Core State Standards**

* CP-2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
* CP-5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

**Overview**

This investigation begins with an introduction to the concept of independent events. Students verify the Multiplication Rule for Independent Events in the context of flipping a coin twice. Then they are told that this rule also can be used to sort out independent events from dependent events. Students discover that if two events *A* and *B* are independent, then so are their complements. Then they calculate several probabilities involving two independent events using the Multiplication Rule for Independent Events. If two events *A* and *B* are dependent, then the probability that *A* will occur is affected by the knowledge that event *B* has occurred. Students investigate the concept of conditional probability informally by comparing a number of conditional and unconditional probabilities from everyday situations. This discussion sets the stage for Investigation 4.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Use the Multiplication Rule for Independent Events to determine whether two events are independent or dependent.
* Use the Multiplication Rule for Independent Events to determine the probability that two independent events occur simultaneously.
* Given two events *A* and *B* from an everyday situation, assess whether is larger than, smaller than, or equal to *P*(*A*).

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 7.3.1** asks students to determine the probability that two events both occur given the events are independent and to decide whether two events are dependent or independent.
* **Exit Slip 7.3.2** asks students to calculate probabilities involving independent events.
* **Journal Entry** asks students to decide if mutually exclusive events can be independent and to support their answer with an explanation.

**Launch Notes**

For this launch, you will need a coin and a container with colored marbles. Adapt the descriptions below to match the actual results when you flip the coin and when you draw a marble from the container.

 Begin by flipping a coin once, and telling students the result. For the purposes of this discussion, suppose the result was a head. Just before you flip the coin a second time, ask students for the probability of getting heads on this second flip. Most students should agree that the probability is still $\frac{1}{2}$, even though you got a head on the first flip.

 Next, put some colored marbles into a container. For the purposes of this discussion, suppose you put 3 red marbles, 4 blue marbles, and 6 cat’s eye marbles into the container. Mix the marbles and without looking draw one out. Without showing what marble you drew from the container, ask students the probability that the marble in your hand is a cat’s eye. Since each marble is equally likely to be drawn from the container, this probability is $\frac{6}{13}$. Show students the marble—let’s say it was a blue marble. Return the marble to the container and mix the marbles. Before drawing out a second marble, ask students the following question: “Given I got a blue marble on the first draw, is it more likely, less likely, or still probability $\frac{6}{13}$. that I draw a cat’s eye?” Students should agree that the probability is still $\frac{6}{13}$. So, in this case, the probabilities associated with the possible outcomes on the second draw are in*dependent* of the outcome on the first draw.

 Draw a second marble and report your results. Suppose this time you drew a cat’s eye. Do not return the marble to the container. Get ready to draw another marble from the container, but before you do, ask students the following question: “Given I just drew a cat’s eye, what is the probability that the next marble I draw will be a cat’s eye?” In this case, the probability has changed – there are now only 12 marbles in the container and only 5 of them are cat’s eyes. So, the fact that you drew out a cat’s eye on the first draw and didn’t put it back into the container before the second draw, changed the probability of drawing a cat’s eye to $\frac{5}{12}$.. So, in this case, the probabilities associated with the possible outcomes on the second draw *depend* on the outcome on the first draw.

**Teaching Strategies**

In the first two situations from the launch discussion—flipping a coin twice and drawing two marbles from a container with replacement—the outcome on the first flip or draw did not influence the outcome on the second flip or draw. In these situations we can say that the two events, “outcome on the first flip/draw” and “outcome on the second flip/draw” are **independent**.

**Independent events**: Events *A* and *B* are independent if the occurrence of one does not change the probability that the other occurs. If two events are not independent, then they are dependent.

**Activity 7.3.1 Independence and the Multiplication Rule** begins with the experiment of flipping a coin twice. Students determine probabilities of two events *A* and *B*, which from the launch discussion they know to be independent. They find that , which is a verification of the Multiplication Rule for Independent Events. Then students use the Multiplication Rule for Independent Events to determine whether other events *A* and *B* are independent.

After students have completed **Activity 7.3.1**, review the Multiplication Rule for Independent Events. Notice that this rule can be used in two ways: (1) to determine whether two events are independent or (2) to calculate the probability that two independent events both occur.

**Multiplication Rule for Independent Events**

If *A* and *B* are independent events, then . Moreover, if for any events *A* and *B*, , then *A* and *B* are independent. If not, then *A* and *B* are dependent.

In **Activity 7.3.1** students are given specific events *A* and *B* and then asked to use the Multiplication Rule for Independent Events to decide whether or not these events were independent. The enrichment challenge below asks students to come up with (1) events *A* and *B* which are independent and (2) events *A* and *B* which are dependent.

**Differentiated Instruction (Enrichment)**

As an extension to Activity 7.3.1, let *S* be the cards from a standard deck of playing cards (with Jokers removed). Ask students to determine (1) a set of events *A* and *B* that are independent and (2) a set of events *A* and *B* that are dependent. In each case express events *A*, *B*, and using set notation. Find *P*(*A*), *P*(*B*), and . Then use the Multiplication Rule for Independent Events to verify that *A* and *B* are independent or dependent.

Sample answers: (1) Let event *A* be the card is a heart and event *B* be the event that the card is not a face card. *P*(*A*) = $\frac{13}{52}=\frac{1}{4}$,  *P*(*B*) = $\frac{40}{52}=\frac{10}{13}$, *P*($A∩B$) = $\frac{10}{52}$ =$\frac{5}{26}$. Multiplication Rule: *P*(*A*) × *P*(*B*) = $\frac{1}{4}$× $\frac{10}{13}$, = $\frac{10}{52}$ = *P*($A∩B$). Hence, *A* and *B* are independent events.
(2) Let event *A* be a numbered card less than 8 and event *B* be a heart numbered card greater than 5. *P*(*A*) = $\frac{28}{52}$ = $\frac{7}{13}$, *P*(*B*) = $\frac{5}{52}$, *P*($A∩B$) = $\frac{2}{52}$ = $\frac{1}{26}$.
Multiplication Rule: *P*(*A*) × *P*(*B*) = $\frac{7}{13}$ × $\frac{5}{52}$ = $\frac{35}{676}$ ≠ *P*($A∩B$). Hence, *A* and *B* are dependent events.

**Exit Slip 7.3.1** may be assigned any time after students have completed **Activity 7.3.1**.

**Activity 7.3.2 Calculating Probabilities Involving Independent Events** presents situations in which events are independent, and asks students to calculate the probability that the two events occur simultaneously. Students also verify that if *A* and *B* are independent events, then  and are also independent events. Then they apply the Multiplication Rule for Independent Events to calculate probabilities.

Question 5 in **Activity 7.3.2** presents the probability distribution of blood types in the U.S. You can find blood type probability distributions for other countries at the following web site: <http://en.wikipedia.org/wiki/Blood_type_distribution_by_country>. Review the solutions to question 5 (d). Check that students understand that you can use the Complement Rule to find the probability of at least one: *P*(at least one) = 1 – *P*(none).

In **Activity 7.3.2** students verified using specific events from rolling a die that if two events are independent then their complements are also independent. Ask students who are up for a challenge to complete the Extension to Activity 7.3.2 that follows.

**Differentiated Instruction (Enrichment)**

Using the Complement Rule, the Multiplication Rule for Independent Events, and one of De Morgan’s Laws show that if *A* and *B* are independent, then  and  are also independent.

We are given *A* and *B* are independent. Hence, . In order to show that  and  are independent, we must prove.

Here is one possible proof:

 De Morgan’s Law

  Complement Rule

  General Addition Rule

  Distributive Law

  Complement Rule

  Multiplication Rule for Independent

 Events

  Factor

  Complement Rule

  Factor

  Complement Rule

**Exit Slip 7.3.2** can be assigned any time after students have completed Activity 7.3.2.

**Activity 7.3.3 Informal Conditional Probability** introduces conditional probability. Students revisit drawing different colored marbles from a jar (part of the launch discussion) and determine several conditional probabilities. Then they are given situations involving two events from everyday situations and are asked if a conditional probability involving these events is larger than, smaller than, or equal to an unconditional probability.

**Group Activity**

Students should work in small groups of 2 - 3 students on **Activity 7.3.3** *Informal Conditional Probability*.

There are two versions of **Activity 7.3.3**. The last situation involves the difference between the following two conditional probabilities:

* *P*(man is tall | man is a professional basketball player)
* *P*(man is professional basketball player | man is tall)

In Activity 7.3.3a students are expected to reason abstractly about the difference based on their understanding of the populations of tall men and basketball players. Activity 7.3.3b gets at the same idea using a drawing of stick figures, allowing students to reason at a more concrete level.

**Differentiated Instruction (For Learners Needing More Help)**

Assign **Activity 7.3.3b** which uses stick figures to help students in questions 3–6.

 In **Activity 7.3.3** students were given specific pairs of events from everyday situations and then asked to decide if a conditional probability involving these events was larger than, smaller than, or the same as an unconditional probability. The enrichment challenge below asks students to come up with their own pairs of events from everyday situations.

**Differentiated Instruction (Enrichment)**

As an extension to Activity 7.3.3, create conditional and unconditional probability pairs of your own that satisfy each of the following:

The conditional and unconditional probability pairs are equal.

The conditional probability is greater than the unconditional probability.

The conditional probability is less than the unconditional probability.

**Journal Entry** can be written after students have completed Activity 7.3.3. Before assigning this journal prompt, explain the meaning of mutually exclusive.

**Mutually exclusive events:** Events that have no outcomes in common. (Also called disjoint events.)

**Journal Entry**

Two events are said to be **mutually exclusive** if they have no outcomes in common. Can mutually exclusive events also be independent events? Explain your reasoning. Look for the following reasoning: Suppose that *A* and *B* are mutually exclusive events and . Then if you know *B* has occurred, *A* can’t occur because *A* and *B* have no outcomes in common. So, *A* and *B* must be dependent events.

**Closure Notes**

The lesson began with examples of independent events from coin tossing and drawing multiple marbles from a container *with replacement*. However, students soon discovered that if the marbles were drawn from the jar *without replacement*, the outcome of the previous draw changed the probabilities associated with outcomes for the next draw. Students then verified with an example related to coin tossing the Multiplication Rule for Independent Events. Then they used this rule to decide whether or not two events were independent and later to calculate probabilities related to independent events. The investigation concluded with comparisons of unconditional and conditional probabilities of events in everyday life. This allowed for an informal discussion of the meaning of conditional probability, which sets the stage for Investigation 4.

**Vocabulary**

**Dependent events:** Events *A* and *B* are dependent if the occurrence of one does change the probability that the other occurs.

**Conditional probability:** The conditional probability of an event *A* given *B* is the probability that *A* will occur given the knowledge that event *B* has already occurred. This probability is written as .

**Independent events:** Events *A* and *B* are independent if the occurrence of one does not change the probability that the other occurs.

**Multiplication Rule for Independent Events:**  If *A* and *B* are independent events, then the probability that they both occur is equal to the product of their individual probabilities.

**Mutually exclusive events:** Events that have no outcomes in common. (Also called disjoint events.)

**Resources and Materials**

Activity 7.3.1: Independence and the Multiplication Rule

Activity 7.3.2: Calculating Probabilities Involving Independent Events

Activity 7.3.3a: Informal Conditional Probability

Activity 7.3.3a: Informal Conditional Probability (scaffolded for students needing more support)

Exit Slip 7.3.1

Exit Slip 7.3.2

Marbles of various colors for Launch