**Activity 7.4.2: Calculating Conditional Probability from a Formula**

**Conditional probability**

The conditional probability of *A* given *B*, written as , can be calculated by dividing the probability that both events occur by the probability that *B* occurs:



Notice that in order to calculate , we need .

1. Consider rolling a pair of dice and observing the sum of the spots on sides landing face up. From Activity 7.2.1, question 4, you determined a probability model for this situation, which appears in Table 1.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Outcomes | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Probability |  |  |  |  |  |  |  |  |  |  |  |

Table 1. Probability model for sum of the spots on a pair of dice.

Let *A* be the event that the sum is greater than 4, and *B* be the event that the sum is even.

a. Find *P*(*A*), *P*(*B*) and 

b. Suppose that you know that *B* has occurred (the sum is even). Does this knowledge make it more likely or less likely that *A* occurs (the sum is greater than 4)? To answer this question use the formula above to determine . Compare your answer to the unconditional probability *P*(*A*), which you calculated in part (a).

c. Suppose, instead, that you know that *A* has occurred (sum is greater than 4). Does this knowledge make it more likely or less likely that *B* occurs (the sum is even)? To answer this question determine . Compare your answer to the unconditional probability *P*(*B*), which you calculated in part (a).

d. Which is larger,  or ? Given *P*(*A*) and *P*(*B*), which you calculated in part (a), why should you not be surprised?

2. Continue using the probability model from Table 1. Let *B* be the event that the sum is even (as it was in question 1). Let *C* be the event that the sum is a multiple of three.

a. Find *P*(*C*) and .

b. Find . Compare this probability to the unconditional probability *P*(*C*). Did the knowledge that *B* had occurred, affect the probability that *C* occurs?

3. Show that if two events *A* and *B* satisfy , then *A* and *B* are independent events.

4. Table 2 gives a probability model for ABO and Rh factor blood types in the U.S. (You saw this table earlier in question 5, Activity 7.3.2.)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | ABO | type |  |
| Rh factor | A | B | AB | O |
| + | 0.357 | 0.085 | 0.034 | 0.374 |
| - | 0.066 | 0.063 | 0.015 | 0.006 |

Table 2. Probability model for blood types in the U.S.

a. Let *E* be the event that a randomly chosen person has ABO blood type A or AB. What is the probability of *E*?

b. Let *F* be the event that a randomly chosen person has Rh+ blood. What is the probability of *F*?

c. What is the probability that a randomly chosen person has blood type A+ or AB+? (This is the event .)

d. Suppose a randomly selected person has ABO blood type A or AB. Does this knowledge make it more likely or less likely that this person has a Rh+ blood? What conditional probability should you calculate? What unconditional probability should you compare it to?

e. Let *G* be the event that a randomly chosen person has ABO blood type B or AB. Let *H* be the event that a randomly selected person has Rh- blood. Compare the unconditional probability *P*(*G*) to the conditional probability . Does knowledge that a person has Rh- blood affect the probability that their ABO blood type is B or AB?

5. Table 3 shows a joint probability model for the average distance driven per week by 12th grade female and male students. (This model was developed using the data from question 8, Activity 7.2.1.)

|  |  |  |
| --- | --- | --- |
| Driving distance | Female | Male |
| None | 0.12 | 0.10 |
| 1 – 10 miles | 0.05 | 0.05 |
| 11 – 50 miles | 0.14 | 0.12 |
| 51 – 100 miles | 0.10 | 0.12 |
| Over 100 miles | 0.09 | 0.11 |

Table 3. Joint probability model for driving and gender.

a. Check to see if the probabilities in this probability model sum to 1.

b. Let *A* be the event that the student drives between 51 and 100 miles per week. Let *F* be the event that that the student is female. At first Anne was not quite sure which conditional probability she wanted. So, she calculated both conditional probabilities and found = 0.2 and . Show the calculations to verify that her results are correct.

c. Anne wants to answer the following question: If a female student is selected at random, how likely is it that she drives between 51 and 100 miles, on average, per week. Which of the two probabilities in (b) answers this question? Explain.

d. Which probability is higher: (1) the probability that male 12th-grade students drive, on average, over 50 miles per week or (2) the probability that female 12th-grade students drive, on average, over 50 miles per week. Answer this question by calculating two conditional probabilities and comparing them. Is this what you expected?