**Activity 7.2.1 Assigning Probabilities**

We begin this activity with the definition of a probability model.

Definition

A probability model is a mathematical description of a random process consisting of two parts: a sample space *S* and a way of assigning probabilities to events.

In this activity, you will create probability models in two ways. In the first case, the outcomes in the sample space are equally likely. In the second case, you will repeat the random process a large number of times and calculate the relative frequency of observing each outcome in the sample space.

*Part I: Equally likely outcomes*

1. Consider the process of flipping a coin. The sample space is *S* = {H, T}. If the coin is a fair coin, then the probability of getting a head should equal the probability of getting a tail; in other words, *P*(H) = *P*(T).

Using rules of probability from Investigation 1, explain why *P*(H) = *P*(T) = ½.

The probability model for the situation of flipping a fair coin is given in Table 1. Notice that the probabilities for the two outcomes are equal and that they sum to 1.

|  |  |  |
| --- | --- | --- |
| Outcome | H (heads) | T (tails) |
| Probability | $$\frac{1}{2}$$ | $$\frac{1}{2}$$ |

Table 1. Probability model for flipping a coin.

2. Consider the random process of flipping a coin twice.

1. Draw a tree diagram that represents the outcomes of the two flips. Then list the possible outcomes in the first row of Table 2.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Outcome |  |  |  |  |
| Probability |  |  |  |  |

Table 2. Probability model for flipping a coin twice.

b. Assuming the coin is a fair coin, each of the outcomes that you listed in Table 2 is equally likely. What probability would you assign to each outcome? To complete the probability model, list these probabilities in the second row of Table 2.

c. What is the sum of the probabilities that you listed in the second row of Table 2?

3. Consider the random process of rolling a die, which has sample space *S* = {1, 2, 3, 4, 5, 6}. If the die is perfectly balanced, then each outcome is equally likely. Complete the probability model in Table 3 by entering the probabilities of each outcome into the second row.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Outcome | 1 | 2 | 3 | 4 | 5 | 6 |
| Probability |  |  |  |  |  |  |

Table 3. Probability model for rolling a die.

4. Consider the random process of rolling a pair of dice, one red and the other green. Figure 1 displays in a table format the outcomes in sample space *S*.



Figure 1. Sample space for rolling pair of dice.

a. Assuming that both dice are perfectly balanced, what probability would you assign to each outcome in *S*? (Note: A single outcome consists of the number on the red die and the number on the green die; for example, red = 2 and green = 5.)

b. The probability of rolling a sum of 2, which occurs when “red = 1 and green = 1,” is the same as your answer to (a). What about the probability of rolling a sum of 3? Write the event of rolling a sum of 3 as the union of two simple events each consisting of individual outcomes from the sample space. Then use the General Addition Rule introduced in Investigation 1 to find *P*(roll a sum of 3).

c. When you have equally likely outcomes, you can find the probability of an event *A* as follows:

 

Explain how to use this formula to calculate *P*(rolling a sum of 4).

d. In Table 4, complete the probability model for rolling a pair of dice and observing the sum of the spots on the sides landing face up. Give both exact probabilities (written as fractions) and decimal approximations (rounded to three decimal places).

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Outcomes | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Probability |  |  |  |  |  |  |  |  |  |  |  |

Table 4. Probability model for the sum of the spots on a pair of dice.

e. What is the sum of the probabilities in your answer to (d)?

***Part II: Using relative frequencies to approximate probabilities***

In Part I you determined how to assign probabilities to outcomes in a probability model using a sample space in which each point is equally likely. However, another way of assigning probabilities to outcomes (or events) is to repeat a random process many, many times and observe the relative frequency of times the outcome (or event) occurs.

Definition

The probability of any outcome (or event) of a random process is the relative frequency of times the outcome (or event) would occur in a very long series of repetitions.

5. a. Roll a die 100 times and record your results in column 2 of Table 5.

|  |  |  |
| --- | --- | --- |
| Number of spots on side landing face up | Frequency | Relative frequency |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |

Table 5. Distribution of number of spots from 100 rolls.

b. Next, calculate the relative frequencies and record them in the last column of Table 5. Here’s how to calculate relative frequency:

Relative frequency = Frequency/(Total number of times random process is repeated)

c. Are the relative frequencies that you entered into Table 5 close to the probabilities you determined in question 3? Recall that probabilities are relative frequencies over a *very long* series of repetitions, not just over 100 repetitions. (In question 7, you will revisit this question using combined data from the class.)

6. Consider the random process of rolling a pair of dice. The sample space consists of the possible sums of the spots on the sides that land face up.

a. Roll a pair of dice 100 times. Record the frequencies in the second column of Table 6. Then calculate the relative frequencies and enter them into the last column.

|  |  |  |
| --- | --- | --- |
| Sum of Spots | Frequency | Relative frequency |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |

Table 6. Distribution of sum of number of spots.

b. Based on your relative frequencies, which outcome is most likely to occur?

7. Probabilities are relative frequencies over the very long run, not just over 100 repetitions of the random process as was requested in questions 5 and 6.

a. Combine the class group results from Tables 5 and 6 into Tables 7 and 8, respectively. We’ll refer to the data in Tables 7 and 8 as the class data.

|  |  |  |
| --- | --- | --- |
| Number of spots on side landing face up | Frequency | Relative frequency |
| 1 |   |   |
| 2 |   |   |
| 3 |   |   |
| 4 |   |   |
| 5 |   |   |
| 6 |   |   |

Table 7. Distribution of number of spots from class data.

|  |  |  |
| --- | --- | --- |
| Sum of Spots | Frequency | Relative frequency |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |

Table 8. Distribution of sum of number of spots from class data.

b. Compare the relative frequencies in Table 7 (from the class data) to the probabilities you determined in question 3. Are the relative frequencies in Table 7 closer to the actual probabilities than your relative frequencies from Table 5 based on only 100 rolls?

c. Compare the relative frequencies in Table 8 (from the class data) to the probabilities you determined in question 4(d). Are the relative frequencies in Table 8 closer to the actual probabilities than your relative frequencies from Table 6 based on only 100 rolls?

d. Based on the relative frequencies in Table 8, which outcome is most likely to occur? Which two outcomes are least likely to occur?

8. Table 9 gives the frequencies on two survey questions given to 12th-grade students. The questions were:

* What is your gender?
* During an average week, how much do you usually drive a car, truck, or motorcycle?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Driving | Female | Male | Total number of students (males and females) | Probability(estimates) |
| None | 1597 | 1344 |  |  |
| 1 – 10 miles | 696 | 611 |  |  |
| 11 – 50 miles | 1795 | 1598 |  |  |
| 51 – 100 miles | 1254 | 1467 |  |  |
| Over 100 miles | 1161 | 1438 |  |  |

Table 9. Distribution of average weekly driving distances for male and female 12th-graders.

a. Complete the last two columns of Table 9. Use relative frequencies (relative frequency = frequency/total number of students) as estimates of the probabilities. Round probabilities to two decimal places.

b. What is the sum of the probabilities in the last column?

c. Which driving distance is most likely? Explain.

d. What is the probability that a randomly selected student drives at least 51 miles per week on average? How did you determine your answer?

e. What is the probability that a student does some driving, on average, each week? How did you determine your answer?