**Unit 2: Investigation 6 (3 Days)**

**Radical Equations**

**Common Core State Standards**

* [REI.A.2](http://www.corestandards.org/Math/Content/HSA/REI/A/2/) Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
* CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.
* CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
* IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
* IF.C.7.A Graph linear and quadratic functions and show intercepts, maxima, and minima.
* IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

**Overview**

This investigation introduces students to contextual situations that can be described by equations with radical expressions. Students learn methods to solve various types of radical equations, to check for extraneous solutions, to understand radical expressions and equations graphically, and to model contextual situations with radical functions.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Solve equations involving one and multiple square root terms.
* Solve radical equations graphically.
* Determine if a solution obtained from an equation-solving process is extraneous.
* Model and solve contextual problems involving radical equations.

**Assessment Strategies: How Will They Show What They Know?**

* **Activity 2.6.1 Radical Functions** introduces students to two real-world contexts that can be described by radical functions.
* **Activity 2.6.2 Solving Radical Equations** provides students opportunities to solve radical equations algebraically and graphically.
* **Activity 2.6.3 Crossing Lake James** provide students opportunities to model contextual situations that focus on minimizing total travel time when a person is using two modes of transportation with different speeds.
* **Exit Slip 2.6.1** asks students to evaluate a radical function and solve a radical equation.
* **Exit Slip 2.6.2** asks students to solve radical equations and check for extraneous solutions.
* **Exit Slip 2.6.3** asks students to model a real-world situation using a radical function.
* **Journal Entry 1** asks students to describe the mathematical behavior of a radical function.
* **Journal Entry 2** asks students to explain how extraneous solutions arise when solving radical equations.

**Launch Notes**

Begin this investigation by showing students a picture of skid marks and explain how a traffic accident reconstructionist measures skid marks to estimate the speed of a vehicle at the instant the driver slammed on the brakes to stop the vehicle. Skid marks occur when a tire moves at fast speed along the ground in a locked position. Ask students to guess what type of mathematical relationship exists between the length of skid marks and the speed of the vehicle immediately before the driver slammed on the breaks. **Activity 2.6.1** begins with a radical function that models the relationship between these two variables.

**Teaching Strategies**

1. **Activity 2.6.1 Radical Functions** introduces students to two real-world contexts that can be described by radical functions: estimating the speed of a vehicle based on the length of skid marks, and modeling growth of children via growth curves. Students graph radical functions and solve radical equations. When students solve radical equations, allow students to reason about the appropriate mathematical step to eliminate the square root in an equation.

Questions 7 and 10 ask students to model data using a function of the form $f\left(x\right)=a+\sqrt{bx}$. Students should complete this task using technology. You may need to model this process for students. Students can fit curves to data via the graphing calculator by employing a trial and error approach, or via Geogebra or Desmos, by employing sliders.

1. **Activity 2.6.2 Solving Radical Equations** provides students opportunities to solve radical equations algebraically and graphically. Using graphing technology, students examine graphical solutions of equations that emerge during the equation-solving process. Students see that in certain cases extraneous solutions arise. These are solutions that satisfy the quadratic equation that emerges when both sides are squared, but do not satisfy the original equation.

**Differentiated Instruction (For Learners Needing More Assistance)**

Provide students simple radical equations to solve so students get additional practice with the procedural steps involved in solving radical equations.

**Differentiated Instruction (Enrichment)**

Provide students problems involving radical equations involving cube root and nth root expressions.

1. **Activity 2.6.3 Crossing Lake James** provide students opportunities to model contextual situations that focus on minimizing total travel time when a person is using two modes of transportation with different speeds. Students must employ the Pythagorean theorem and the $distance=rate × time $relationship to create a function to model the time it takes to travel from point A to point P (see figure below) and use the total time it takes to travel from point A to point C.



**Group Activity**

Question 13 in **Activity 2.6.3** asks student to create a problem situation similar to Crossing Lake James that involves two modes of transportation with different speeds. This is a challenging problem that is appropriate for a group activity.

**Differentiated Instruction (Enrichment)**

Question 13 in **Activity 2.6.3** asks student to create a problem situation similar to Crossing Lake James that involves two modes of transportation with different speeds. Prompt students to check whether their problem situation has an optimal solution that incorporates both modes of transportation. Students can use sliders in Geogebra or Desmos to explore how different speeds or distances impact the optimal solution of their model of their problem situation.

**Journal Entry 1** asks students to describe the mathematical behavior of a radical function. This should be assigned after **Activity 2.6.1.**

Students can describe an increasing radical function that increases at a fast rate initially but then increases at a slower and slower rate as *x* increases. The function always increases but the graph of the function approaches a horizontal line.

**Journal Entry 2** asks students to explain how extraneous solutions arise when solving radical equations. This should be assigned after **Activity 2.6.2.**

Students can state that an extraneous solution arises when we square both sides of an equation to generate a quadratic equation. Then, when solving the quadratic equation, one of the solutions to the quadratic equation is not a solution of the original radical equation.

**Vocabulary**

Extraneous solution

Radical equation

Square root equation

**Resources and Materials**

**All three activities in this investigation should be completed.**

* Activity 2.6.1 Radical Functions
* Activity 2.6.2 Solving Radical Equations
* Activity 2.6.3 Crossing Lake James
* Birth to 24 months: Boys Length-for-age and Weight-for-age percentiles <http://www.cdc.gov/growthcharts/data/who/grchrt_boys_24lw_100611.pdf>
* Birth to 36 months: Girls Length-for-age and Weight-for-age percentiles <http://www.cdc.gov/growthcharts/data/set2clinical/cj41l068.pdf>
* TI-83 Graphing Calculator
* Geogebra
* Desmos