**Unit 5: Investigation 8 (3-4Days)**

**Ellipses and Hyperbolas**

**Common Core State Standards**

G-GPE.3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant; or given foci and directrices.

**Overview**

This is an optional unit for STEM intending students. We extend the concept of locus to the other conic sections, the ellipse and the hyperbola. Both are constructed through paper folding activities and then analyzed algebraically. Applications of ellipses are introduced as is the concept of eccentricity.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Write the equation of an ellipse or a hyperbola in standard position given the foci and the constant sum or difference.
* Given the equation of an ellipse or a hyperbola, find the foci and determine the eccentricity

**Assessment Strategies: How Will They Show What They Know?**

* **Exit slip 5.8.1** has students find the equation of an elliptical orbit.
* **Exit slip 5.8.2** has students use the locus definition to show that points lie on a hyperbola.
* **Journal Entry** has students explain the differences between the equations of ellipses and hyperbolas

**Launch Notes**

Start by showing a three-minute video about Johannes Kepler and his discovery that the orbits of the planet around the sun are elliptical. <https://www.youtube.com/watch?v=qDHnWptz5Jo>. The video shows how an ellipse may be drawn with a pencil and a string anchored at the two foci.

**Teaching Strategies**

**Activity 5.8.1 Ellipses in the Coordinate Plane** introduces the ellipse as the locus of points the sum of whose distances from two fixed points is constant. Student begin by verifying that a set of points has this property. They observe that the foci lie on the major axis and are introduced to the equation of an ellipse in standard position with major axis along the *x-*axis:

After students complete this activity, you may demonstrate the locus definition dynamically with the GeoGebra file: Ellipse\_as\_locus.ggb.

**Activity 5.8.2 Constructing an Ellipse through Paper Folding** provides a hands-on method to generate an ellipse from the set of its tangent lines. The property of the perpendicular bisector as the locus of points equidistant from the endpoints of a segment is used to justify this construction method.

You may use the file Ellipse\_paper-folding\_demo.gbb to show how the paper folding construction works.

In **Activity 5.8.3 Deriving the Standard Equation of an Ellipse** students complete a formal derivation of the equation of an ellipse with foci at (–*c*, 0) and (*c,* 0) and major axis length *a*. The algebra in this derivation is quite complex, so most of the steps are shown. This activity can be used as a model for the question in **Activity 5.8.5** where students write a similar derivation for the standard equation of a hyperbola.

**Differentiated Instruction (For Learners Needing More Help)**

Students for whom the algebra in Activity 5.8.3 seems to formidable may instead derive the equation with specific value of the parameters, for example *a* = 5, *b* = 4, and *c* = 3.

**Activity 5.8.4 Ellipses in Our World** introduces students to a variety places ellipses are found. The orbits of Earth and Halley’s comet are modeled with ellipses and the contrast in their eccentricities is noted. In addition the area formula: *A* = π*ab* is introduced and justified informally through the horizontal stretch and vertical stretch transformations that were briefly introduced in Unit 1.

Following Activity 5.8.4 you may give students Exit Slip 5.8.1.

**Activity 5.8.5 Hyperbolas** introduces students to the last of the four conic sections studied in this unit. Throughout this activity hyperbolas are compared and contrasted with ellipses. First the hyperbola is constructed through a paper folding exercise similar to the one used in Activity 5.8.2 for ellipses. Then students construct the locus definition for hyperbola and use it to derive the standard equation.

In conjunction with this activity you may demonstrate the locus definition of hyperbola with the file Hyperbola\_as\_locus.ggb.

Following Activity 5.8.5 you may give students Exit Slip 5.8.2.

**Differentiated Instruction (Enrichment)**

Have students derive equations for the ellipse and hyperbola with foci on the *y­-*axis. Compare and contrast these equations with the “standard” equations with foci on the *x*-axis.

In **Activity 5.8.6 Eccentricity of the Conic Sections** students revisit the locus definition of parabola and extend it by requiring that the ratio of the distance from the point to the focus (*ep*,0) and the distance to the directrix (*x* = –*p*) is a positive constant *e*. When *e* > 1 the figure is a hyperbola, when *e* = 1 it is a parabola, and when 0 < *e* < 1 it is an ellipse. Students use the GeoGebra file ctcoregeomACT586 to change the value of *e* and observe the effect on trace.

**Group Activity**

Students may play a matching game to help review properties of conic sections. Use the template provided. Cut 32 cards from each template. Shuffle the cards and give five to each player and place the remaining cards in a stack. On each turn the player draws a card. They lay out a pair that match (e.g. *e* > 1 and “eccentricity for a hyperbola”). If they can’t play, they discard one card and draw another card. First player to play all their cards wins.

**Journal Entry**

Compare and contrast the standard equations for ellipses and hyperbolas. Look for students to see the connection between the equations and the locus definitions of these two conic sections.

**Closure Notes**

Explain that the circle, ellipse, parabola, and hyperbola, are all called conic sections because they can be found when a plane is passed through a conical surface. This can be illustrated with a concrete model or a with video. For example, <https://www.youtube.com/watch?v=iJOcn9C9y4w> gives a five-minute demonstration of how the conic sections are formed. Cross-sections of three dimensional figures will be studied in more detail in Unit 6.

**Vocabulary**

asymptote

conic secion

directrix (of conic section)

ellipse

focus (of ellipse, hyperbola)

hyperbola

major axis

minor axis

transverse axis

**Coordinate Geometry Equations**

Ellipse in Standard Position:

Hyperbola in Standard Position:

Focus-Directrix Definition of Conic Section (Directrix *y = –*1, Focus (0,*e*):

**Resources and Materials**

Wax paper or parchment paper for Activities 5.8.2 and 5.8.5.

Geogebra files for teacher demonstration:

Ellipse\_as\_locus.ggb

Ellipse\_paper\_folding\_demo.ggb

Hyperbola\_as\_locus.ggb

GeoGebra file for student use: ctcoregeomACT586.ggb.

Cards for matching game: Unit\_5\_Investigation\_8\_matching\_game.docx

Videos:

<https://www.youtube.com/watch?v=qDHnWptz5Jo> for launch.

<https://www.youtube.com/watch?v=iJOcn9C9y4w> for closure.

Activity 5.8.1 Ellipses in the Coordinate Plane

Activity 5.8.2 Constructing an Ellipse through Paper Folding

Activity 5.8.3 Deriving the Standard Equation of an Ellipse

Activity 5.8.4 Ellipses in Our World

Activity 5.8.5 Hyperbolas

Activity 5.8.6 Eccentricity of the Conic Sections