**Unit 5: Investigation 6 (3 Days)**

**Inscribed Angles and Intercepted Arcs**

***CCSS:***

G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

G-C 4. (+) Construct a tangent line from a point outside a given circle to the circle.

**Overview**

In **Investigation 6** we show that an inscribed angle in a circle is equal in measure to one-half the measure of the central angle that intercepts the same arc. A special case is Thales’ Theorem that the angle inscribed in a semicircle is a right angle. We also show that the opposite angles of a cyclic quadrilateral are supplementary. An optional activity (addressing standard C-4+) will have students construct tangents to a circle from an external point.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Know and apply the Inscribed Angle Theorem and its corollaries
* Understand the method of proof by cases
* Construct tangents to a circle from an outside point (STEM intending students only)

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 5.6** requires a proof that two inscribed angles intercepting the same arc are congruent.
* **Journal Entry** asks students to describe the proof by cases used for the Inscribed Angle Theorem

**Launch Notes**The teacher begins with an experiment on the projected computer using either Geometer’s Sketchpad or Geogebra that shows measures of two angles that intercept the same arc, one with vertex at the center of the circle and the other with vertex on the circle. These angles are called **central angles** and **inscribed angles** respectively. See file Central\_and\_inscribed\_angle.ggb or Central\_and\_inscribed\_angle.gsp. Students should come up with a conjecture about the relationship between the measures of the angles.

**Teaching Strategies**

In **Activity 5.6.1 Inscribed Angles and Intercepted Arcs** students build on the launch by measuring with inscribed angles and the central angles which intercept the same arc. They formulate the conjecture which becomes the Inscribed Angle Theorem: The measure of an angle inscribed in a circle is one-half the measure of the central angle that intercepts the same arc. The proof requires three cases 1) the case where the center of the circle is on one side of the inscribed angle 2) the case where the center of the circle is in the interior of the inscribed angle and 3) the case where the center of the circle lies in outside the inscribed angle.

Note: We have chosen to state the Inscribed Angle Theorem in terms of the measures of the inscribed angle and the central angle that intercepts the same arc. Another approach is to define the angular measure of an arc as the same as its corresponding central angle. In that case the theorem can be expressed as “The measure of an angle inscribed in a circle is one-half the measure of its intercepted arc.” We chose not to use this approach since GeoGebra measures only the length of an arc, not its angular measure. Furthermore the proof of the theorem highlights the relationship between the central angle and the inscribed angle.

**Differentiated Instruction (For Learners Needing More Help)**

Remind students that two radii of a circle form two central angles, typically one that measures less than 180° and one that measures more than 180°. They intercept a minor arc and a major arc respectively. If there is any ambiguity in naming the angles, the one that measures more than 180° may be designated as “reflex angle *AOB*,” etc. They also need to understand that when the two radii are collinear, they form two straight angles, each measuring 180°. They will need this concept in the proof of Thales’ Theorem, below.

Exit Slip 5.6 may be given at any time after completion of Activity 5.6.1.

**Journal Entry**

This unit has a proof that must be done by breaking it into cases. Why did we need to do this? Look for students to explain that Case 1, when the center lies on one side of the angle, is the easiest case to prove and that once it is proved it can be used to prove the other cases.

In **Activity 5.6.2 Thales’ Theorem** students prove that the angle inscribed in a semicircle is a right angle as a corollary of the theorem introduced in Activity 5.6.1. This is a straight-forward proof which should be accessible to students with little scaffolding. Students then apply the theorem in several exercises.

In **Activity 5.6.3 Cyclic Quadrilaterals** students show that the opposite angles of a cyclic quadrilateral are supplementary. The lesson begins with small group experiments. Students examine inscribed quadrilaterals. They measure the angles of the quadrilaterals and come up with a conjecture. They then develop a proof by drawing a diagonal of the quadrilateral and use the inscribed angles to show that the angles are supplementary. They also explore which special quadrilateral are cyclic.

**Group Activity** Have students work in groups on the proofs in Activity 5.6.2 and Activity 5.6.3. Then have one student from each group present their proof to another group.

**Activity 5.6.4 Constructing Tangents to a Circle from an Outside Point** is optional for students who intend to pursue STEM related careers. It asks students to construct the two tangents to a circle from a point outside the circle.

**Differentiated Instruction (Enrichment)**

Assign optional Activity 5.6.5 described below.

**Activity 5.6.5 General Relationships between Arcs and Angles** provides students an opportunity to generalize relationships between arcs and angles. This activity introduces students to thinking about the possible cases. How many ways could intersecting lines intersect a circle and intercept an arc or arcs? They will look for what happens when the intersection of the lines is outside the circle or inside the circle as well as very special situations where one or both of the sides of the angle formed are tangents.

**Closure Notes**

Use the GeoGebra file regular\_pentagram\_in\_circle.gbb for a brief class discussion to wrap up the investigation. Explain that points *B,* *C*, *D*, *F*, and *G* are equally spaced around the circle. Have students figure out the measures of $∠GAB$ and $∠GDB$ and justify their results.

**Theorems**

**Inscribed Angle Theorem**: The measure of an angle inscribed in a circle is one-half the measure of the central angle that intercepts the same arc.

**Thales’ Theorem:** An Angle inscribed in a semi-circle is a right angle

**Cyclic Quadrilateral Theorem:** The opposite angles of a cyclic quadrilateral are supplementary.

Note: The converses of Thales’ Theorem and the Cyclic Quadrilateral Theorem are also true and are mentioned in Activities 5.6.2 and 5.6.3.

**Construction:** To construct the tangent segments to a circle from a point outside the circle (+)

**Vocabulary**

central angle

cyclic quadrilateral

inscribed angle

intercepted arc

**Resources and Materials**

Activity 5.6.1 Inscribed Angles and Intercepted Arcs

Activity 5.6.2 Thales’ Theorem

Activity 5.6.3 Cyclic Quadrilaterals

Activity 5.6.4 Constructing Tangents to a Circle from an Outside Point (Optional)

Activity 5.6.5 General Relationships between Arcs and Angles (Optional)

Geometer’s Sketchpad or GeoGebra for experiments.

Central\_and\_inscribed\_angles.ggb

Central\_and\_inscribed\_angles.gsp

ctcoregeomACT565a

ctcoregeomACT565b

regular\_pentagram\_in\_circle.ggb for closure

Geometry construction tools plus protractor for measuring angles