**Unit 5: Investigation 4 (3 Days)**

**Tangents to Circles**

***CCSS:***

G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

G-C 4. (+) Construct a tangent line from a point outside a given circle to the circle.

G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

**Overview**

In **Investigation 4** we see an informal proof of the fact that a tangent to a circle is perpendicular to a radius drawn to the point of tangency. Two right triangles are congruent if their hypotenuses and one pair of legs are congruent, a theorem that will be used to prove that tangent segments drawn from an external point to a circle are congruent. One activity will revisit the equation of a circle in the coordinate plane and use the slopes of perpendicular lines to find the equation of a line tangent to a circle at a given point

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Explain why a tangent to a circle is perpendicular to the radius drawn to the point of tangency.
* Understand and use the Hypotenuse Leg Congruence Theorem for Right triangles.
* Explain and apply the Tangent Segments Theorem
* Use coordinate geometry to solve problems involving circles and tangents

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 5.4** asks students to apply the Radius-Tangent Theorem.
* **Journal Entry** asks students to explain how the Hypotenuse Leg Congruence Theorem is used to prove the Tangent Segments Theorem.

**Launch Notes**

The teacher can launch this section with a brief experiment using Geometer’s Sketchpad or Geogebra. You may use the file Experimenting\_with\_Secants.ggb for this purpose. Construct a circle. Use the center of a circle and a point on the circle to construct a radius. Select another point on the circle and join the two points on the circle with a segment. Measure the angle formed by this segment and the radius. Notice what happens to the measure of the angle as you move closer to the point of tangency. Introduce the vocabulary secant, tangent, point of tangency.

**Teaching Strategies**

In **Activity 5.4.1 Exploring Circles and Secants** students measure an angle formed by a radius and a secant through the endpoint of the radius on the circle and another point on the circle to make a conjecture about the relationship between a radius and a tangent to the endpoint of the radius. Without using the word “limit,” this activity suggests that the tangent line may be viewed as the limit of the secant as two points on the circle approach each other.

**Differentiated Instruction (Enrichment)**

As an extension to **Activity 5.4.1** some students may explore what happens to the measure of $∠$ *ACB* as point *C* moves closer to point *B* on the circle.

**Activity 5.4.2** **The Radius-Tangent Theorem** provides an indirect proof that the tangent to a circle is perpendicular to the radius drawn to the point of tangency and conversely. In order to do that we need to show that the shortest segment from a point to a line is perpendicular to the line. You may want to use this activity for whole class instruction.

**Differentiated Instruction (For Learners Needing More Help)**

Some students may need extra help understanding the indirect proof in question 2b of Activity 5.4.2.

**Exit Slip 5.4** may be assigned after Activity **5.4.2.**

In **Activity 5.4.3** **Two Theorems Involving Right Triangles** students use transformations to prove the Hypotenuse-Leg Congruence Theorem. This activity then asks students to examine some illustrations to determine if the HL congruency applies. It then uses one of the examples to introduce the Tangent Segments Theorem.

**Differentiated Instruction (Enrichment)**

Ask students to formulate and prove a theorem about similar triangles that is analogous to Hypotenuse Leg Congruence Theorem.

The Tangents Segment Theorem is then applied to situations (such as pulleys) when lines are externally tangent to two circles in **Activity 5.4.4 Applications of Tangents.**

**Activity 5.4.5** **Tangents in the Coordinate Plane** revisits the equation of a circle in the coordinate plane and uses the slopes of perpendicular lines to find the equation of a line tangent to a circle at a given point.

**Group Activity**

Have students work in groups of three. The first student picks a point in the coordinate plane to be the center of a circle. The second student chooses a radius for the circle. The third student picks another point in the plane. The group then decides whether this point lies inside, outside or on the boundary of the circle. Students then switch roles and repeat the exercise.

**Closure Notes**

Review the ways we have learned to prove that two triangles are congruent. Recall that SSA in general is not as set of sufficient conditions for triangles to be congruent, but the Hypotenuse-Leg Congruence Theorem is a special case, applicable only to right triangle.

**Journal Entry**

Explain how congruent triangles are used to prove the Tangent Segments Theorem. Which Congruence Theorem is needed for this proof? Look for students to identify the Hypotenuse-Leg Congruence Theorem.

**Theorems**

**Shortest Distance Theorem:** The shortest segment joining a point to a line is the perpendicular segment.

**Radius-Tangent Theorem:** A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

**Radius-Tangent Converse:** If a line is perpendicular to the radius of a circle at a point on the circle, then it is tangent to the circle.

**HL Congruence Theorem:** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

**Tangent Segments Theorem**: If two tangents are drawn to a circle from the same point outside the circle, the segments from that point to the circle are congruent.

**Vocabulary**

point of tangency

radius

secant

tangent

tangent segment

**Resources and Materials**

Protractors for Activity 5.4.1

GeoGebra file: Experimenting\_with\_Secants.ggb

Activities:

 Activity 5.4.1 Exploring Circles and Secants

 Activity 5.4.2 The Radius-Tangent Theorem

 Activity 5.4.3 Two Theorems Involving Right Triangles

 Activity 5.4.4 Applications of Tangents

 Activity 5.4.5 Tangents in the Coordinate Plane