**Unit 5: Investigation 1 (2 Days)**

**Circles in the Coordinate Plane**

***Common Core State Standards***

* G-GPE.1**.** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
* G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

**Overview**

In this investigation, students are formally introduced to the concept of circles. We will define a circle as the locus of all points in a plane that are the same distance from a given point (the center). Students will first examine circles centered at the origin. We will use the Pythagorean theorem and the distance formula to derive an equation for all such circles. We will then use translation and dilation to discuss circles not centered at the origin. Finally, we will use the process of completing the square to find the center and radius of any circle in the form:

 .

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Define the term “locus of points.”
* Use the Pythagorean theorem to derive the formula for circles centered at origin.
* Complete the square to find the center and radius of a circle given by an equation in the form:
.

**Assessment Strategies: How Will They Show What They Know?**

* Exit slip 1 asks students to find the equation of a circle given its center and radius and vice versa.
* Exit slip 2 asks students to complete the square to find the center and radius of a circle.
* Journal prompt asks students to explain why there is an *x*2 and a *y*2 in the equation of a circle.

**Launch Notes**

We will begin this unit with an object almost all students will be familiar with—the Ferris Wheel. It offers an excellent way to discuss circles and locus of points in a real-world context.

You may want to show students a rendering of the soon-to-be-tallest-in-the-world Ferris Wheel that will be built on Staten Island, NY: <http://newyorkwheel.com/category/blog/>.

**Teaching Strategies**

You may initiate a discussion of the locus definition of circle by constructing a circle on an overhead projector. Use pieces of spaghetti or any other equal-sized segments such as straws or cutout pieces of construction paper. Draw a point to represent the center and then arrange the segments around the point. At this point, do not draw in the circle. Now, for emphasis, draw a point at the end of each of the segments and connect those points. Ask students if all the points are an equal distance from the center point. Explain that these points are called a locus and define locus as a collection of items with the same property. In this case, our locus is the set of the points in a plane that are all a fixed distance from the center point. This is exactly how we are going to define a circle:

 Circle – the locus of points in a plane that are the same distance from a given point (the center).

Ask students what we call this fixed distance. Most should recognize this distance as the radius of the circle. It is interesting to note that we use the term radius to refer both to the actual segment that connects the center of the circle to a point on the circle, as well as, the actual length of that segment.

In **Activity 5.1.1** Circles with Center at the Origin the distance formula or the Pythagorean Theorem is used to obtain the formula for all such circles. Display a circle centered at the origin.

*r*

Now explain that we are going to draw a radius and we will label it *r*.

We will also label the point on the circle (*x, y*). At this point, we want to draw a right triangle with the radius of the circle as our hypotenuse. Because we named the point (*x, y*) and because our circle is centered at the origin, the lengths of the two legs are the absolute values of *x* and *y*. When squared, it does not matter whether *x* and *y* are positive or negative: *x*2 and *y*2 are always non-negative.

*r*

*y*

*x*

**Differentiated Instruction (for Learners Needing More Help)**

Some students may have difficulty seeing why the lengths of the sides of the triangle are *x* and *y*. You may want to pick values for *x* and *y* (i.e., 2 and 5) and show that the base of the triangle formed is 2 units from the center and the other leg is 5 units from the center.

Students should recognize that we have a right triangle with legs of lengths |*x*| and |*y*|and a hypotenuse of length *r*. Using the Pythagorean theorem we obtain the equation: . Note that we can also use the distance formula between the points (0,0) and (*x*,*y*) to obtain the same equation.

Point out that the special case where the radius is equal to 1 is called a **unit circle** and its equation subsequently is

Note that it is important that students recognize that the equation of a circle is not a function. This might be a nice place to review the definition of function from Algebra 1. Students should recall that, in order to be a function, no *x*-value can have more than one corresponding *y*-value. If we want to graph a circle using a graphing calculator we must graph two functions: and

Ask students to discuss with their neighbors whether that equation would represent all the points on the circle. It is also important to point out that all circles of the form may be considered images of the unit circle under a dilation centered at the origin with scale factor equal to *r*.

Given a circle with equation show that the distance from the origin to a point inside the circle is less than the radius whereas the distance from the origin to a point outside the circle is greater than the radius.

**Activity 5.1.2** **Circles Anywhere in the Coordinate Plane** examines circles not centered at the origin. Students will see how we can use the Pythagorean theorem again, as well as dilation and translation, to get the general equation of a circle:

Make sure students understand and recognize the subtraction expressions. Thus, if the equation of the circle was , the center would be located at (4,5). So, a minus sign in the parentheses leads to a positive coordinate and a plus sign would lead to a negative coordinate. This is where using the distance formula instead of the Pythagorean theorem to derive the equation is better. The minus signs are built in to the distance formula already.

In this activity each student may be assigned a letter corresponding to a point on each of the given circles. They can verify that their point is 5 units from the origin, and compare their result with those of students who were assigned a different letter.

At the end of the first day you may give **Exit Slip 5.1.1**, which asks students to find the center and radius of a circle given its equation.

**Activity 5.1.3** **Completing the Square to Find Equations of Circles** reviews completing the square with students in order to solve problems that ask for the center and radius of circles given by equations in the form: . A good problem to start with is: . The center comes out to be (2,–3) and the radius is 4.

**Differentiated Instruction (For Learners Needing More Help)**

Some students may not have gotten to completing the square in their Algebra 1 course and may need extra time acquiring this skill. Or, you may decide to defer this skill to Algebra 2 and skip this activity.

(Completing the square was introduced in Unit 8 of the Connecticut Core Algebra 1 curriculum. If your students did not get to that Unit in Algebra 1, you may want to defer this activity to the Algebra 2 course.)

At the end of the second day you may give **Exit Slip 5.1.2**, which asks students to complete the square to find the center and radius of a circle.

**Differentiated Instruction (Enrichment)**

Some students may be fascinated with the extension of the Pythagorean Theorem to the work done by Fermat who examined the equation *xn* + *yn* = *zn* and theorized that for non-zero integers, *x, y, z*, and *n*, there are no non-zero solutions when n gets larger than 2. This is often referred to as Fermat’s Last Theorem. Find more info here: <http://mathworld.wolfram.com/FermatsLastTheorem.html>

At the end of the second day you may give **Exit Slip 5.1.2**, which asks students to find the center and radius of a circle by completing the square.

**Journal Prompt:** Why is there an *x*2 and a *y*2 in the equation of a circle? Look for students to explain how the distance formula is used to find the radius.

**Vocabulary**

locus of points

circle

center

completing the square
radius

**Coordinate Geometry Equations**

**Circle with Center at the Origin and Radius *r*:**

**Circle with Center (*h*, *k*) and Radius *r*:**

**General Equation of a circle:**

**Resources and Materials**

The New York Wheel Ferris Wheel:

<http://newyorkwheel.com/category/blog/>

History of the Ferris Wheel:

<http://www.hydeparkhistory.org/newsletter.html>

<http://inventors.about.com/od/tstartinventions/ss/theme_park.htm#step-heading>

Ferris Wheel on Unit Circle:

<http://maine.edc.org/file.php/1/tools/FerrisWheelUnitCircle.html>

Activities:

 Activity 5.1.1 Circles with Center at the Origin

 Activity 5.1.2 Circles Anywhere in the Coordinate Plane

 Activity 5.1.3 Completing the Square to Find Equations of Circles (optional for students who did not complete Unit 8 of the Algebra 1 curriculum)