**Activity 7.4.4 – Testing Claims on Population Means**

*Statistical inference* is the process of using sample statistics to make conclusions about population parameters. This activity shows how a sample mean can be used to assess whether a claim about the value of a population mean is reasonable.

**Using Sample Evidence to Test Claims about an Unknown Population Mean**

Data from Facebook’s monthly users indicate that Facebook users spend an average of 20 minutes per day on Facebook. Suppose a classmate says that the mean amount of time Facebook users at your school spend on Facebook is *more than* 20 minutes per day. It is impossible to get information from every Facebook user at school, so you gather data from a sample of Facebook users to assess whether your friend is correct. Suppose you obtain a random sample, Sample 1, shown below, with the following sample statistics.

**Sample 1**

$$n=30$$

$$\overbar{x}=22 min$$

$$s=8.8 min$$

1. Does this sample lead you to conclude that your friend is correct – that the mean amount of time Facebook users at your school spend on Facebook is more than 20 minutes per day? Explain.
2. Suppose you find a second random sample of 30 Facebook users shown below. Would this sample lead you to a different conclusion?

**Sample 2**

$$n=30$$

$$\overbar{x}=40 min$$

$$s=6.2 min$$

The situation above can be addressed by a *hypothesis test*. In a hypothesis test we use sample statistics to test a claim made about the value of a population parameter. In doing so, we assume that the value of the parameter equals a specific value and then assess the likelihood of observing the sample results under this assumption. Hypothesis tests involve uncertainty since we never actually know that value of the population parameter.

**Randomization Hypothesis Test**

A *randomization hypothesis test* assesses the likelihood of a sample statistic from a randomization distribution. A *randomization distribution* is a distribution of sample statistics randomly obtained via simulations from a population with a fixed parameter.

**How Much Money Do You Spend on Music?**

A media magazine claims that high school students in the U.S. spend an average of $20 on music each month. Suppose we obtain a random sample of 20 students from a local high school and determine the amount of money each student spends on music each month.

Random Sample: Monthly Amounts Spent on Music, *n* = 20

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 14 | 14 | 15 | 20 | 8 | 2 | 7 | 6 | 24 | 8 |
| 10 | 12 | 7 | 9 | 14 | 11 | 15 | 25 | 9 | 10 |

This sample has a mean of $\overbar{x}=\$12$ and a standard deviation of $s=\$5.86$. Does this sample provide evidence that the mean amount of money all students at the local high school spend on music each month is less than $20?

To answer this question, we do something unusual: we assume the population mean $μ$ is 20. This is the *hypothesis* we test. Then, we find the probability of obtaining a sample mean $\overbar{x}$ of 12 or less, assuming the population mean $μ$ is 20. We do this via a randomization distribution of sample means.

**Constructing a Randomization Distribution of Sample Means** $\overbar{x}$

We can construct a randomization distribution of sample means by:

* Transforming the original sample to a modified sample with mean equal to the hypothesized mean,
* Randomly sampling (with replacement) from the modified sample to generate a randomization distribution of sample means $\overbar{x}$.

The first step is to transform the original sample to a modified sample with mean 20. To do this, each value in the sample must be transformed in a consistent way.

1. How can we modify each value in the original sample to create a new sample with a mean of 20?

The modified sample is shown below. 8 was added to each value in the original sample to create the modified sample. The modified sample has the same variability (standard deviation) as the original sample but a different mean.

Modified Sample: Monthly Amounts Spent on Music, *n* = 20

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 22 | 22 | 23 | 28 | 16 | 10 | 15 | 14 | 32 | 16 |
| 18 | 20 | 15 | 17 | 22 | 19 | 23 | 33 | 17 | 18 |

We treat this modified sample as the “population” with mean $μ=20$. The dotplot of the modified sample is shown below.

Modified Sample = “Population”

$$μ=20$$



We can create a randomization distribution of sample means by generating random samples of size *n* = 20 from this population, calculating sample means, and constructing a distribution of sample means. We sample with replacement to generate random samples.

The table below shows the modified sample (“population”) with each value labeled with a number between 1 and 20.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Label | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Amount ($) | 22 | 22 | 23 | 28 | 16 | 10 | 15 | 14 | 32 | 16 |
| Label | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Amount ($) | 18 | 20 | 15 | 17 | 22 | 19 | 23 | 33 | 17 | 18 |

1. *Random Sample 1 –* Randomly select 20 numbers between 1 and 20, with replacement. For each random number, identify the corresponding value in the “population”. Calculate the sample mean.
2. *Random Sample 2 –* Randomly select 20 numbers between 1 and 20, with replacement. For each random number, identify the corresponding value in the “population”. Calculate the sample mean.
3. Create a dotplot of sample means using all the sample means from your class.



The randomization distribution above was formed under the assumption that the population mean is $μ=20$. The variability in the sample means is due to sampling variability.

1. Use technology to determine the mean and standard deviation of sample means in the randomization distribution.
2. According to the randomization distribution, what is the probability of obtaining a random sample with a mean of $12 or less?

A *P*-value is the probability of obtaining a sample statistic as extreme as the one observed assuming the population parameter is equal to a specific value.

* When a *P*-value is less than 5%, we say the sample statistic is *statistically significant*. This means that that it is likely that the statistic did not emerge due to chance alone. We reject the assumption about the population parameter.
* When a *P*-value is greater than or equal to 5%, we say the sample statistic is *not statistically significant*. This means that the sample statistic could have occurred solely due to chance. We do not reject the assumption about the population parameter.
1. Is the observed sample mean statistically significant?
2. We tested the hypothesis that the mean amount of money students at the local high school spend on music each month is $μ=\$20$. What can we conclude about this population mean? Explain.

**Time Spent on Homework**

The Organization for Economic Co-operation and Development (OECD) reported that, in 2012, 15-year old U.S. teenagers spent an average of 6 hours per week completing homework. A teacher decided to examine how students at her school compared to this national average. She randomly surveyed 24 students at her school and found the following sample data and sample statistics.

Original Sample: Weekly Amount of Time Spent on HW, *n* = 24

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 | 6 | 5 | 6 | 9 | 8**Original Sample**$$\overbar{x}=7.5$$$$s= 3.65$$ |
| 7 | 14 | 7 | 8 | 7 | 14 |
| 0 | 2 | 10 | 3 | 9 | 9 |
| 7 | 12 | 7 | 10 | 7 | 12 |

Does this sample provide evidence that mean amount of time students at her school spend completing homework each week is greater than 6 hours?

We can conduct a randomization test as follows:

* Assume the mean amount of time students at her school spend completing homework each week is $μ$ = 6 hours.
* Transform the original sample to a modified sample with mean $μ$ = 6.
* Randomly sample from the modified sample to construct a randomization distribution of sample means from random samples of size *n* = 24.
* Find the probability of observing a sample mean as extreme as the one found.
* State a conclusion about the population mean.

We will test the hypothesis that the population mean is 6. To do so, we create a modified sample by subtracting 1.5 from each value in the original sample. The modified sample is shown below.

Modified Sample: Weekly Amount of Time Spent on HW, *n* = 24

**Modified Sample**

**(“Population”)**

$$μ=6$$

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| -0.5 | 4.5 | 3.5 | 4.5 | 7.5 | 6.5 |
| 5.5 | 12.5 | 5.5 | 6.5 | 5.5 | 12.5 |
| -1.5 | 0.5 | 8.5 | 1.5 | 7.5 | 7.5 |
| 5.5 | 10.5 | 5.5 | 8.5 | 5.5 | 10.5 |

The following graph shows a randomization distribution of 400 sample means from random samples of size *n* = 24. The random samples were obtained by sampling with replacement from the modified sample.

Randomization Distribution of Sample Means



The randomization distribution above was formed under the assumption that the population mean is $μ=6$. The variability in the sample means is due to sampling variability.

1. The observed sample mean is $\overbar{x}=7.5$. According to the randomization distribution, what is the probability of obtaining a random sample with a mean of 7.5 hours or more?
2. Is the observed sample mean statistically significant?
3. We tested the hypothesis that the mean amount of time students at the teacher’s high school spend completing homework each week is $μ=6$ hours. What can we conclude about this population mean? Explain.