**Activity 5.8.5 Hyperbolas**

You’ll need the following materials for this activity:

1 piece of wax paper or parchment paper

Compass

Pen/Pencil

1. Follow these steps to carry out the construction.

1. On your piece of wax paper, use your compass to construct a fairly large circle. (Be sure

to make the radius small enough so that the entire circle is contained on the wax paper.

1. Label the center point of your circle. Label this point *F*1.
2. Plot and label another point in the exterior of this circle. Label this point *F*2. (Note: this is the key difference between this construction and the one you made in Activity 5.8.2 for the ellipse.
3. Plot approximately 20-25 points *on the circle*. (Just draw dots to represent these points).

Label *any one* of these points as *P*.

1. Take the wax paper and fold it so that point *P* lies on top of point *F*2. Crease sharply.
2. Repeat step (5) above for all the other points you plotted on the circle (back in step 4).

That is, treat each point on the circle as another “point *P*.” Simply fold each “point *P*” on

the circle to point *F*2. *Be sure to crease sharply each time!*

2. Describe the figure formed by the intersection of all the creases.

We will now examine some properties of the figure created by the creases.



3. The figure at the right shows one of the points *P* that was folded onto *F*2.

Draw the ray $\vec{F\_{1}P}$ so that it intersects the fold line at *H*.

The fold line is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ bisector of $\overbar{PF\_{2}}$. It passes through *M*, the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of $\overbar{PF\_{2 }}$

4. Every point on $\overleftrightarrow{HM }$is equidistant from points and .

This means that *HP* = \_\_\_\_\_.

5. We will now show that the difference of the distances from the two foci to *H* is constant, no matter which point on the circle was chosen as point *P.*

a. Since the radius of a circle *never changes*, it is said to be .

b. By segment addition, *F*1*H* = \_\_\_\_\_ + \_\_\_\_\_\_\_

c. But we saw in question 4 that *HP* = \_\_\_\_\_. Therefore *F*1*H* = \_\_\_\_\_ + \_\_\_\_\_\_\_.

d. Solve the above equation for *F*1*P,* the radius of the circle.

Therefore *F*1*H*  –*F*2*H* is the same for all points *H*.

6. The curve traced by the points *H* described in question 3 is only one of the two branches of the curve generated by the paper folding construction. When *P* is on the other side of the circle the ray $\vec{F\_{1}P}$ no longer intersects the fold line. Instead we need to draw the ray $\vec{PF\_{1}}$ to locate a point “*J*” where it intersects the fold line.

Show that for all such points *J*, the quantity *F*2*J*  –*F*1*J* is constant.

7. The figure formed by the paper folding construction is called a **hyperbola.** Fill in the blanks to create its definition:

Definition: A **hyperbola** is the locus of points the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of whose distances from two fixed \_\_\_\_\_\_\_\_\_(called the **foci**) is a \_\_\_\_\_\_\_\_\_\_\_\_.

8. Study the above definition. In what ways is it like the definition of *ellipse*? In what ways is it different?



9. Here is a hyperbola in standard position in the coordinate plane.

The foci are located at (–*c*, 0) and (*c,* 0).
The two branches of the hyperbola cross the *x*-axis at (*a*, 0) and (–*a*, 0). The *x*-axis is the **transverse axis** of the hyperbola.
The points (0, *b*) and (0, –*b*) lie on the *y-*axis.

The rectangle shown by the dashed line contains the four points (*a*, 0), (–*a*, 0), (0, *b*), and (0, –*b*). The dotted lines containing the diagonals of this rectangle are the **asymptotes** to the hyperbola. You will learn more about asymptotes in a future course. In a future course it will also be shown that $c^{2}=a^{2}+b^{2}$.

Now with all that information, you can show that an equation for this hyperbola in standard position is

$\frac{x^{2}}{a^{2}}–\frac{y^{2}}{b^{2}}=1.$ The derivation of this equation is very similar to the derivation of equation of an ellipse in standard position, which was shown in Activity 5.8.3. Use that as an example to derive the equation $\frac{x^{2}}{a^{2}}–\frac{y^{2}}{b^{2}}=1.$