**Activity 5.1.3 Completing the Square to Find Equations of Circles**

**Equations of circles in factored form**

For each of the following equations, identify the center and the radius.

1. *x*2 + y2 = 42 Center: \_\_\_\_\_\_\_ Radius = \_\_\_\_\_\_\_\_\_

2. *x* 2 + y2 = 25 Center: \_\_\_\_\_\_\_ Radius = \_\_\_\_\_\_\_\_\_

3. (*x* – 4)2 + (y – 2)2 = 49 Center: \_\_\_\_\_\_\_ Radius = \_\_\_\_\_\_\_\_\_

4. (*x* – 4)2 + (y + 3)2 = 82 Center: \_\_\_\_\_\_\_ Radius = \_\_\_\_\_\_\_\_\_

**Equations of circles that are not in factored form**

You can also identify the center and radius of circles with equations like:

*x* 2 – 8x + 16 + *y*2 + 6*y* + 9 = 64

Separate the equation into an “*x*-portion” and a “y-potion”. Then factor each portion.

|  |  |  |
| --- | --- | --- |
| Equation | “*x*-portion” | “y-portion” |
| *x* 2 – 8 *x* + 16 + *y*2 + 6*y* + 9 = 64 | *x* 2 – 8*x* + 16 | *y*2 + 6*y* + 9 |
| Factored: | (*x* – 4)2 | (*y* + 3)2 |

Now, in the factored equation, it is easier to identify the center and the radius.

*x*2 – 8*x* + 16 + *y*2 + 6*y* + 9 = (*x* – 4)2 + (*y* + 3)2 = 82

*Put the following equations in factored form. Then identify the center and radius of each circle.*

5. *x*2 – 10*x* + 25 + *y*2 + 8*y* + 16 = 25

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center: \_\_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_

6. *x*2 + 12*x* + 36 + *y*2 – 2*y* + 1 = 49

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center: \_\_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_

7. *x*2 – 14*x* + 49 + *y*2 – 4*y* + 4 = 100

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center:\_\_\_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_

8. *x*2 + 2*x* + 100 + *y*2 + 20*y* + 1 = 4

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center: \_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_

9. *x*2 + 14*x* + *y*2 + 2*y* + 50 = 9

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center: \_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_

**Equations that are not easily put in factored form**

You can also identify the center and radius of circles with equations like:

*x*2 – 8*x* + *y*2 + 6*y* = 39

Separate the equation into a “x-portion” and a “y-potion”. Complete the square for each portion.

|  |  |  |
| --- | --- | --- |
| Equation | “*x*-portion” | “y-portion” |
| *x*2 – 8*x* + *y*2 + 6*y* = 39 | *x*2 – 8*x* | *y*2 + 6*y* |
|  | *x*2 – 8*x* + | *y*2 + 6*y* + |
| *x*2 – 8*x* ***+* (-4)2** + *y*2 + 6*y* ***+* 32**= 39 ***+* (-4)2 + 32** | **Note:** Add the same numbers on both sides of the equation. |  |
| *x*2 – 8*x* + 16 + y2 + 6*y* + 9 = 39 + 16 + 9 |  |  |
| (*x* – 4)2 + (*y* + 3)2 = 64 |  |  |

Now, with the equation in factored form, it is easier to identify the center and the radius.

*x*2 – 8*x* + *y*2 + 6*y* = 39 becomes (*x* – 4)2 + (*y* + 3)2 = 82

*Put the following equations in factored form. Then identify the center and radius of each circle.*

10. *x*2 + 10*x* + *y*2 + 8y = 23

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center: \_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_

11. *x*2 – 12*x* + *y*2 – 2*y* = 12

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center: \_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_

12. *x*2 + 14*x* + *y*2 – 4*y* = 11

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center: \_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_

13. *x*2 + 2*x* + *y*2 + 20*y* = 20

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center: \_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_

14. *x*2 – 14*x* + *y*2 + 2*y* = 31

Factored: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Center: \_\_\_\_\_\_\_\_ Radius: \_\_\_\_\_\_\_\_\_