**Activity 2.2.5 Completing the Square**

Another method for solving a quadratic equation is called “Completing the Square”, but first, some practice with “perfect squares.”

Square the following binomials by multiplying them out using the distributive property. Then try to describe a pattern in the trinomial that is the result from squaring thebinomial.

|  |  |
| --- | --- |
| Square of a binomial |  Perfect Square Trinomial |
| $(x+1)^{2}$= |  |
| $(x+2)^{2}$= |  |
| $(x+3)^{2}$= |  |
| $(x+4)^{2}$= |  |
| $(x+5)^{2}$= |  |
| $(x+6)^{2}$= |  |

2. Based on the patterns you observed above, describe how to fill in the missing number so that t $x^{2}+16x+\\_\\_\\_$ is a perfect square trinomial.

3. Fill in the missing numbers to make a true statement:

a. $(x-7)^{2}$= $x^{2}-14x+\\_\\_\\_$ b. $(x+\\_\\_\\_)^{2}$= $x^{2}+18x+81$

c. $(x+\\_\\_\\_)^{2}$= $x^{2}+10x+\\_\\_\\_$ d. $(x-\\_\\_)^{2}$= $x^{2}-20x+\\_\\_\\_$

e. $(x+\\_\\_\\_)^{2}$= $x^{2}+3x+\\_\\_\\_$ f. $(x-\\_\\_)^{2}$= $x^{2}-7x+\\_\\_\\_$

4. Solve the equations by taking the square root of both sides of the equal sign. Why do you need to take the negative as well as the positive value of the principal square root of the number? (refer back to p. 11 Activity 2.2.4)

a.  b. 

c.  d. 

In the following examples, the squared expression is not alone. What happens if you square root both sides before you isolate the squared expression? What happens if you isolate the squared expression on one side of the equal sign?

e.  f. 

g.  h. 

i. 

j. In full sentences, describe the steps you took to solve equations when the variable is inside a squared expression:

What kind of answer do you obtain if after taking the square root on both sides of the equal sign; do you get a negative number inside the square root?

k.  l. 

 When a negative number is inside the radical sign, the answer is not a real number.

What kind of answer do you obtain when the number inside the square root is positive, but not a perfect square? Be sure to simplify the answer as much as possible. Do not use decimal approximations.

m.  n. 

p. When the number inside the radical is not a perfect square, the two solutions are \_\_\_\_\_\_\_\_\_\_\_\_\_\_ numbers.

You have practiced how to solve a quadratic equation of the form $\left(x+some number\right)^{2}=some other number$. Any quadratic equation can be put in the form $\left(x+c\right)^{2}=d$ by completing the perfect square trinomial, factoring the trinomial so it is of the form $\left(x+c\right)^{2}$ that is the square of a binomial.

5. Now solve the equation in Column 2, by following the steps in Column 1:

a.

|  |  |
| --- | --- |
| Column 1: Steps for solving a quadratic equation | Column 2: Follow the steps to solve  by “completing the square”. |
| Step 1: Isolate the terms that contain an x by using the Addition Property of Equality |  |
| Step 2: If the coefficient of the quadratic term is not 1, change it to 1 by dividing both sides of the equation by the coefficient of the quadratic term.  |  |
| Step 3: Divide the coefficient of the linear term in half and square it. Add the result to both sides of the equation, using the Addition Property of Equality. This creates a perfect square trinomial, which can be factored into the square of a binomial. |  |
| Step 4: Factor the perfect square trinomial you just created into the square of a binomial. |  |
| Step 5: Take the square root of both sides of the equation and simplify. Remember the + and the negative sign when you simplify the absolute value or the square root of a squared expression. |  |
| Step 6: Solve for *x* in each equation, and simplify the expressions. |  |

5.b.

|  |  |
| --- | --- |
| Column 1: Steps to Complete the Square | Column 2: Follow the steps to solve by “completing the square”. |
| Step 1: Isolate the *x* terms that contain an x by using the Addition Property of Equality |  |
| Step 2: If the coefficient of the quadratic term is not 1, change it to 1 by dividing both sides of the equation by the coefficient of the quadratic term. |  |
| Step 3: Divide the coefficient of the linear term in half and square it. Add the result to both sides of the equation, using the Addition Property of Equality. This creates a perfect square trinomial, which can be factored into the square of a binomial. |  |
| Step 4: Factor the perfect square trinomial you just created into the square of a binomial. |  |
| Step 5:Take the square root of both sides of the equation and simplify. Remember the + and the negative sign when you simplify the absolute value or the square root of a squared expression. |  |
| Step 6: Solve for *x* in each equation, and simplify the expressions. |  |

5.c. Here is an example where the coefficient of the squared term is not “1”.

|  |  |
| --- | --- |
| Column 1: Steps to Complete the Square | Column 2: Follow the steps to solve by “completing the square”. |
| Step 1: Isolate the *x* terms that contain an x using the Addition Property of Equality |  |
| Step 2: If the coefficient of the quadratic term is not 1, change it to 1 by dividing both sides of the equation by the coefficient of the quadratic term. |  |
| Step 3: Divide the coefficient of the linear term in half and square it. Add the result to both sides of the equation, using the Addition Property of Equality. This creates a perfect square trinomial, which can be factored into the square of a binomial. |  |
| Step 4: Factor the perfect square trinomial you just created into the square of a binomial. |  |
| Step 5: Take the square root of both sides of the equation and simplify. Remember the + and the negative sign when you simplify the absolute value or the square root of a squared expression. |  |
| Step 6: Solve for *x* in each equation, and simplify the expressions. |  |

6. Solve the quadratic equations by completing the square. Some answers will be irrational. Some equations will have no real number solution. Be sure to write out the two solutions to the equation in simplest form.

a.  b. 

c.  d. $x^{2}+12x-45 $

e.  f. 

 g.  h. x2 – 6x = -7

7. In the Investigation 1, Activity 2.1.5 How do Quadratic Functions Behave? you were asked to use a calculator to estimate the x intercepts of the graph of a quadratic function that was given to you in vertex form:. Now you can find the x intercepts algebraically.

a. Sketch a graph of the function

Please show your work for the following

1. What are the solutions to the equation ? (If any)
2. What are the coordinates of the x intercepts of the graph of the function  ?
3. What is the y intercept of  ?
4. What is the vertex of 
5. Graph  showing the coordinates of the x intercepts, (if any), the y intercept, and the vertex.



7. b. Sketch a graph of the function

Please show your work for the following:

1. What are the solutions to the equation ? (If any)
2. What are the coordinates of the x intercepts of the graph of the function 
3. What is the y intercept of 
4. What is the vertex of 
5. Graph  showing the coordinates of the x intercepts, (if any), the y intercept, and the vertex.



8. a. Now try a new problem:

Sketch a graph of the function

Please show your work for the following:

1. What are the solutions to the equation ? (use completing the square.)
2. What are the coordinates of the x intercepts of the graph of the function 
3. What is the y intercept of 
4. What is the vertex of  (hint: when you “complete the square” , and before you take the square root of both sides of the equation in part ‘i’ , the equation looks similar to a function in vertex form if you move the constant on the side with the squared term and have 0 on one side of the equal sign.)
5. Graph  showing the coordinates of the x intercepts, (if any), the y intercept, and the vertex.

Note: you can use the technique “completing the square” to transform a quadratic in standard form into vertex form.  is equivalent to $f\left(x\right)=\left(x+3\right)^{2}-16$ .



8. b. Sketch a graph of the function

Please show your work for the following:

1. What are the solutions to the equation ? (Use any method.)
2. What are the coordinates of the x intercept(s) of the graph of the function 
3. What is the y intercept of 
4. What is the vertex of  (hint: the axis of symmetry is halfway between the x intercepts)
5. Graph  showing the coordinates of the x intercept(s), the y intercept, and the vertex.
6. Write the function  in vertex form by completing the square.

