**Unit 5: Investigation 3 ( 2-3 Days)**

**Logarithmic Scales**

**Common Core State Standards**

F.LE.4 For exponential models, express as a logarithm the solution to where *a, c* and *d* are numbers and the base *b* is 2, 10 or *e;* evaluate the logarithm using technology.

**Overview**

This investigation focuses on applications of logarithmic functions. Students will see why scientists need logarithmic scales in addition to linear scales to better display data graphically. When the range of data values is very large, say from 1 to 1010, students will see that scaling should be considered in order to facilitate ease in computing and to make it easier to understand the problem at hand. Students will use logarithmic scales in applications that measure the magnitude of earthquakes, the pH levels of acidity or alkalinity in a substance, the brightness of stars, surface area measurements in geography, and sound in decibels. The launch has them make a log *y* vs *x* graph after noting that a graph of output, *y* vs input, *x* does not display the data well. The focus of the activity is that we need to get a graph that provides a better picture of the data values. We need a graphical display that makes it clearer to see ALL the data. In Investigation 5, an activity especially for STEM -intending students, students will examine the relationship between a log *y* vs *x* graph that is linear and it equation in terms of *x* and log *y* and the associated exponential function f(*x*)=ab*x* in terms of *x* and *y*.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Use properties of logarithms to solve equations.
* Use a logarithmic scale.
* Use formulas that have logarithms in them.
* Apply their knowledge of logarithmic scales to real world applications.

**Assessment Strategies: How Will They Show What They Know?**

**Exit Slip 5.3**. Students will be asked to describe when a logarithmic scale might be the better choice over a linear scale and to solve two applied problems, one involving decibels and one on earthquakes.

**Journal Prompt 1** Explain why a you might use a logarithmic scale instead of a linear scale and provide at least one real world example where a logarithmic scale is used and explain why it is used.

**Activity 5.3.1 Can we Eat the Chicken?** focuses on why we might we better served by a logarithmic scale when we are graphing datawhoserange of data values is very large.

**Activity 5.3.2 Earthquakes** begins with the logarithmic scale sometimes used to measure the magnitude of earthquakes, the Richter scale. Magnitudes of earthquakes measure the energy released at the source of the earthquake. A 9.0 earthquake releases a trillion times as much energy as a 1.0 barely discernable if discernable at all earthquake, a very large range of possible values. The magnitude of an earthquake is essentially the log of the amplitude of the earthquake as measured on a seismograph, so the numbers are converted to a range from 1 to 9. Students will find magnitudes for various earthquakes as well as compare an earthquake’s magnitude relative to other known earthquakes.

**Activity 5.3.3** **Basic or Acidic** has students use the pH scale which is a logarithmic scale. A strongly acidic solution can have 100 million million or 10E14 times more hydrogen atoms than a strongly basic solution and the hydrogen ion and hydroxide concentrations in everyday solutions can vary over that wide range. Students will find pH levels or hydrogen ion concentrations for known substances, such as bleach, ammonia and coffee.

**Activity 5.3.4** **Measuring** **Sound Intensity** looks at the intensity of sound and its measurement. Students explore some problems that will make them aware of the dangers to their hearing when exposed to very loud sounds.

**Activity 5.3.5 Problems Using logarithmic Scales** contains additional application problems and some practice solving equationswith logarithmic expressions or needing a logarithmic function.

**Launch Notes**

Pose the following situation to the class.

Suppose you go over to a friend’s house and they have raw chicken on their counter. When you ask why the chicken is on the counter and not in the refrigerator, your friend’s mom is upset for she says she forgot to refrigerate the chicken and must now discard it. You ask why, and Mrs. Lee responds that poultry is a high risk food for food poisoning because it is a moist food and it was a warm day. You went on the web and one source stated a single bacterium could multiply to 2,000,000 in just 7 hours and a second source stated under ideal conditions you could have 17,000,000 in just 12 hours. Distribute **Activity 5.3.1**. Students can work in pairs or small groups. Verify that for the first source the bacteria double about every 20 minutes and for the second source that the bacteria would double about every 30 minutes.

**Group Activity 5.3.1** Students could work in pairs or groups making just one pair of graphs per group**.** The observations made regarding the graphs when completed should be written and each recorder for a group could report out in large class mode so a class discussion could take place.

The teacher might remind them that Activity 5.1.4 had them consider multiplying by using the exponents of the powers rather than using the large numbers that were the powers. Let us

explore this concept further. Stress to students that when the range of data values is very large, say from 1 to 1010, scaling should be considered in order to make computations reasonable and to make it easier to understand the problem at hand. A linear scale will make it difficult to “see” all the data. In our graph for instance, when we used a linear scale on the vertical axis we were unable to distinguish between all the early data points. They all appeared to lie on the x-axis. Logarithmic scales are used in measuring typically when there is a huge range in measurement values and exponential behavior is suspected. They are used in measuring the magnitude of earthquakes, the pH levels of acidity or alkalinity in a substance, the brightness of stars, surface area measurements in geography, and measuring sound in decibels to name a few of the more familiar applications. A logarithmic scale is one in which the units on an axis are the exponents or logarithms of a base number and it is typically used when the increase or decrease in value on that axis is exponential. If only one axis uses a logarithmic scale then it is called a semi-log chart. If both axes use a logarithmic scale, then it is called a log-log chart. After students have redrawn the graph with a vertical axis that has log (number of cells), it should look linear which for now students can just observe. An activity especially for STEM intending students will examine this observation further in investigation 5 and make a connection between a linear graph of log *y* vs *x* and a linear equation in variables log *y* and *x* and an exponential function’s graph *y* vs *x* and its equation.

**Teaching Strategies**

Show one of the three videos--- The National Geographic one or the Canadian one are only 3 minutes. The WatchKnowLearn site has many good ones ranging from 2 – 11 minutes. See Resources at the end of this overview. The one put out by the Canadian Research Council is especially instructive. A logarithmic scale is a scale used when the range of quantities is very large; by using logarithms, linear numbers are turned into orders of magnitude. Although we may think of the Richter Scale when we hear the word earthquake, Earthquake scientists usually use the Moment Magnitude Scale to measure earthquakes. It measures the size of earthquakes in terms of the energy released. The numbers attached to earthquakes using either scale are almost the same until magnitude 7. The newer scale discriminates better between more serious earthquakes. It is more accurate for earthquakes above 7 on the Richter scale. The Moment Magnitude Scale was developed in the 1970s to succeed the 1930s-era [Richter magnitude scale](https://en.wikipedia.org/wiki/Richter_magnitude_scale). Even though the formulae are different, the new scale retains the familiar continuum of magnitude values defined by the older one. The MMS is now the scale used to estimate magnitudes for all modern large earthquakes by the [United States Geological Survey](https://en.wikipedia.org/wiki/United_States_Geological_Survey).

The Richter scale is determined by a logarithmic function R = log (A/A0) that is used to measure the magnitude of earthquakes. The magnitude of an earthquake is related to how much energy is released by the quake. Instruments called seismographs detect movement in the earth; the smallest movement that can be detected shows on a seismograph as a wave with [amplitude](javascript:void(0)) A0.

A – the measure of the amplitude of the earthquake wave

A0 – the amplitude of the smallest detectable wave

The intensity of an earthquake will usually measure between 2 and 10 on the Richter scale. Fortunately no 10s have occurred and people generally are not aware of under 2 magnitude earthquakes when they occur. Any earthquakes registering below a 5 are fairly minor; they may shake the ground a bit, but are seldom strong enough to cause much damage. Earthquakes with a Richter rating of between 5 and 7.9 are much more severe, and any quake above an 8 is likely to cause massive damage. (The highest rating ever recorded for an earthquake is 9.5 during the 1960 Valdivia earthquake in Chile.) **Activity 5.3.2** explores finding magnitudes for various earthquakes as well as making some comparisons between earthquakes. It can be used in class or for homework as long as the formula used has been discussed in class. The formula is a simplified version of formulas used.

**Differentiated Instruction (Enrichment)** Students could read Aftershock by Elizabeth Kolbert in the June 2015 Smithsonian Magazine. It references a legal battle and scientific debate about predicting earthquakes referring to the 2009 earthquake in L’Aquila, Italy that killed hundreds. Seismologists were found guilty for not predicting an earthquake and were given 6 year sentences in jail. An interesting class debate could be held.

**Activity 5.3.3** examines problems that use the logarithmic scale, the pH scale, to deal with the large range of hydrogen ion concentrations. This is a good activity to borrow a chemistry teacher for a bit especially if time permits the teacher or a small group of students to work with a chemistry teacher and have a small demonstration to measure the pH of a few substances using pH test paper or indicator sticks where the color change is then matched to a key or a pH meter with a probe.

pH is the level of the acidity or alkalinity of a substance. Values of pH range from 0 to 14. For acid, the range is from 0 to 7 pH; levels for alkaline substances are from 7 to 14 pH. A pH level of 7 is called neutral. An example of neutral pH is distilled water. The formula used to measure pH is where [H+] is the hydrogen ion concentration, measured in moles per liter. It is always easier to use a logarithmic scale than having to write all the zeros. Each one unit change in the pH scale corresponds to a tenfold change in hydrogen ion concentration. **Activity 5.3.3** explores finding pH levels and hydrogen ion concentrations for some common substances. This activity can be skipped in the interests of time especially if time is short and students have not studied pH in chemistry already.

**Differentiated Instruction (Enrichment)** Students could work with the chemistry teacher and measure the pH of some common substances for the rest of the class as a whole class small experiment. The class could then use the pH equation to find the hydrogen ion concentrations.

Weber and Fechner studied psychological responses to intensity changes in stimuli. Their law, the Weber-Fechner Stimuli Law stated that that the perceived change was proportional to the logarithm of the intensity change. The threshold of being able to hear some sound is assigned a relative intensity of 1 and it can range to 10,000,000,000, 000 at which point a human ears hurts having reached the threshold of pain. When it is very quiet it is easy to notice a small change in sound intensity but on a busy city street the same increase would probably not be noticed. So the decibel scale is logarithmic, it measures order of magnitude changes. Sound is measured bels, a very large unit named after Alexander Graham Bell, so instead we use decibels and the formula: Noise level in decibels= , where I0 is the intensity of a sound that can barely be heard. Assume or 10-16 watts/cm2 . The expression I/I0 gives the relative intensity of sound, in the same way we compared the magnitude of earthquakes to a reference value. **Activity 5.3.4** looks at the intensities of various situations given the levels and vice versa. Activity 5.3.4 finds dB for various noises so student can appreciate the danger to hearing in exposure to loud noises. **Exit slip 5.3** can be used any time after Activity 5.3.4.

**Differentiated Instruction (For Learners Needing More Help)**  Students may find a review of scientific notation helpful, working through each step when using the dB equation and justifying with the Law of Exponents, a logarithm property or clarifying where the Distributive Property is being used.

**Activity 5.3.5** includes a few application problems as well as some practice solving equations where logarithms are in the equation or needed to find the solution.

**Journal Prompt 1** Explain why a you might use a logarithmic scale instead of a linear scale and provide at least one real world example where a logarithmic scale is used and why it is used. Students might say when they have data values that range from very small to extremely large. They might also mention that exponential behavior is suspected. We use a logarithmic scale to measure earthquakes, or sound because we have values from the very small to extremely large.

**Closure Notes**

Almost halfway into this unit , it might be a good time to step back and talk about how logarithms permit us to put very large numbers on a human friendly scale.

106 or 1 million seconds is ? days

109 or 1 billion seconds is ? days or ?? years

1012 or 1trillion second is ? days or ?? years

1023 is about the number of stars in the universe

1080 is about the number of atoms in the observable universe

But log 106 is 6, log 109 is 9, log 1080 is 80 and the numbers 6, 9, and 80 are much easier to get ones hands on. When dealing with a series of multiplications, logarithms help “count” them just liked addition counts for us when effects are added. Logarithms are exponents.

Lastly, The video pbskids.org/dragonflytv/show/extremesounds.html provides a good summary of the sound intensity work done in Activity 5.3.4. It can be followed up by some questions:

Summarize the video.

What information did it convey?

Why and when do we use logarithmic scales?

What scale(s) is/are used for an earthquake and what do the numbers mean?

What does pH measure?

**Vocabulary**

Amplitude

Decibels

Intensity (earthquakes, sound)

Magnitude

Richter scale

pH levels

Linear scale

Logarithmic scale

**Resources and Materials**

**If time is an issue not all of the problems in Activities 5.3.2 – 5.3.5 need be assigned or you can omit Activity 5.3.3.**

Activity 5.3.1 Can we Eat the Chicken?

Activity 5.3.2 Earthquakes!

Activity 5.3.3 Basic or Acidic

Activity 5.3.4 Measuring Sound Intensity

Activity 5.3.5 Problems Using logarithmic Scales

**Video resources for earthquakes:**

* Video.nationalgeographic.com/video/101-videos/earthquakes-101 well done less than 3 minutes.
* Goggle search: Earthquake videos for teachers and one in this subsection is [www.watchknowlearn.org](http://www.watchknowlearn.org) which has videos in the measuring earthquakes category and a 2 minutes one The Science of earthquakes is very good
* The National Research Council of Canada, How Do We Measure Earthquakes is excellent, very instructive and only 2 minutes long.

**Video for sound**

* pbskids.org/dragonflytv/show/extremesounds.html

June 2015 Smithsonian Magazine ,”Aftershock”, pages 36 – 43 by Elizabeth Kolbert

TI 84 graphing calculator or similar technology

Graph paper

Internet access

http:/earthquake.usgs.gov/learn/topics/mag\_vs\_int.php