**Activity 5.5.5 Fitting an Exponential Function by Linearizing the Data**

**Section I**

The table below lists the number *y* (in millions) of cell-phone subscribers in the US from 1988 to 1997, and *x* is the number of years since 1987.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| *y* | 1.6 | 2.7 | 4.4 | 6.4 | 8.9 | 13.1 | 19.3 | 28.2 | 38.2 | 48.7 |

One last approach to find a good fit that is often used is to transform the data, to see if the transformed data values form a linear pattern. Once the data have been linearized, find the regression line and the correlation coefficient of the transformed data. Lastly, undo the transformation by using the appropriate inverse function to obtain the exponential function that fits the original data values.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ln *y* |  |  |  |  |  |  |  |  |  |  |

1. First take the ln y or the log y. We will use the ln y.
2. Enter the *x* values in List 1 and place the values ln *y* values in List 2. Make a scatter plot. Do the data look fairly linear?
3. Find the equation of the linear regression line and the correlation coefficient *r*.
4. Your equation reads but remember *y* really is ln *y* and you have concrete numbers for *m* and *b*.

So

Use the left and right sides of your equation as equal exponents of *e*.

So

When we plug in the numbers for m and b, we get

Based on the multiplication rule,

What does *y* equal?

1. Now enter the original *y* values in List 3 and perform exponential regression using List 1 and List 3. You should obtain the same equation as the one you obtained in question 4 and the same correlation coefficient as the one you obtained in question 3. By the way, this is how your calculator gets the exponential regression.
2. Below you will find a table relating the weight of a golden retriever puppy and its age in days. (Source: <http://www19.homepage.villanova.edu/alice.deanin/courses/Mat7310/exponential.htm>)

|  |  |
| --- | --- |
| **Age (days)** | **Weight (kg)** |
| 0 | 1.48 |
| 10 | 1.93 |
| 20 | 2.5 |
| 30 | 3.18 |
| 40 | 4.09 |
| 50 | 5.23 |
| 60 | 6.82 |
| 70 | 8.64 |

1. Enter the data in your graphing calculator and graph it. It should look exponential.
2. Create a list for logarithm base 10 of the weight values.

|  |  |
| --- | --- |
| **Age** |  |
| 0 |  |
| 10 |  |
| 20 |  |
| 30 |  |
| 40 |  |
| 50 |  |
| 60 |  |
| 70 |  |

1. Plot log (weight) vs age on your graphing calculator and place a sketch below. Label and scale the axes appropriately.



1. The graph should be linear. Is it?
2. Find the linear regression equation and the correlation coefficient.
3. Now undo the transformation as we did in part b above. Check to see that an exponential regression for weight vs age yields the same equation.

**Section II – Linearizing Power Functions (Optional)**

Suppose the data may be fit by a power function . We found that that bird data (wing spans and weight) were related by a power function in Activity 4.2.1. Taking the natural logarithm of both sides of a power function yields . Let . This gives the form:

which shows that when plotting , power data will appear linear.

To find the power equation use each side as the exponent of *e*: *e*lny = *e*lna+mlnx; so y = *e*lna*e*mlnx;

y= a(*e*lnx)m so y = axm.

1. You can verify this by plotting the data from the bird problem again. Use technology and enter in L1 and L2 the original data if you did not save it and then in L3 and L4 take the ln of L1 and L2. Graph L3 and L4 and find the linear regression for the data should look linear. Write down the linear regression equation and the correlation coefficient *r*.

Remember .

1. Now take the power regression for the data in L1 and L2. Write down the power regression equation and the correlation coefficient *r*. The *r* values should be the same.
2. Take your equation in question 1 and transform it. It should be the same as the equation you got in question 2.
3. Where do you think the r value for the power function comes from?