**Activity 5.1.1: United States Census**

Every 10 years, the United States conducts a census to count the number of people in the country. The first Census occurred in 1790. In Algebra 1, when you first learned about exponential functions, you saw that population growth often can be represented by exponential models. For now, assume that the population growth of the United States is exponential with respect to time ($t$).

**Part 1 – Create a Model**

According to the U.S. Census, the population in 1790 was 3.929 million. By 1800, the population had grown to 5.308 million.

1. What value of $t$ corresponds to the year 1790? Why is this a good choice?
2. What value of $t$ corresponds to the year 1800?
3. What does the variable $t$ represent?
4. Keep in mind that population is a function of time and you are assuming that the function is exponential. Using the given data points (0, 3.929) and (1, 5.308) where the input variable represent time in decades since 1790,, write two equations of the form
$$P=a∙e^{kt}$$

and solve them to find values for $a$ and $k$.

1. What is the value of $a$? What does *a* represent in the situation?
2. What is the value of $k$? How does this number relate to the situation?
3. Write an exponential function relating population ($P$) and time ($t$).

**Part 2 – Use the Model**

1. Using the function you created in Part 1 number 7, predict the population in the year 1830.
2. Using the function from Part 1 number 7, predict the population in 1880.
3. Which do you think will be a more accurate prediction, the 1830 population or the 1880 population? Why?

**Part 3 – Evaluate the Model**

Based on just two data points you created an exponential model for population growth in the United States. Was it safe to assume that population growth in the U.S. would follow an exponential trend? Examine more data to see if it was a good assumption.

1. Graph your exponential model from part 1 and the following data on the same coordinate system.

|  |  |
| --- | --- |
| **Year** | **Population (in millions)** |
| 1810 | 7.240 |
| 1820 | 9.638 |
| 1830 | 12.866 |
| 1840 | 17.069 |

Explain whether or not your model appears to give a good prediction of future population.

1. Add the following to the data set and graph.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| **Year** | **Population (in millions)** |
| 1850 | 23.192 |
| 1860 | 31.443 |
| 1870 | 38.558 |
| 1880 | 50.156 |
| 1890 | 62.948 |

 |

|  |  |
| --- | --- |
| **Year** | **Population (in millions)** |
| 1900 | 75.996 |
| 1910 | 91.972 |
| 1920 | 105.711 |
| 1930 | 122.775 |
| 1940 | 131.669 |

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Explain whether or not the original model appears to be a good fit for the data? If the graph does not appear to fit an exponential model over the domain from 1790 on, describe the shape of the graph.

1. Add the following to the data set and graph.

|  |  |
| --- | --- |
| **Year** | **Population (in millions)** |
| 1950 | 150.697 |
| 1960 | 179.323 |
| 1970 | 203.185 |
| 1980 | 226.546 |
| 1990 | 248.710 |

Explain whether or not the original model appears to be a good fit for the data? If the graph does not appear to fit an exponential model, describe the shape of the graph.

**Part 4 – Evaluate the Assumptions**

In the initial problem description, you were told to assume that the population growth was exponential and to create a mathematical model. Evaluate this assumption. Specifically, was this a good assumption or did the model eventually breakdown? Give a detailed reply and include possible reasons for the trends in population growth.

**Part 5 – Further Evaluate the Assumptions (Extension)**

Quite often, math text book writers create data that will “nicely” model a mathematical topic.

1. Is the data given in this activity real data from the United States Census or has it been modified to fit the purpose of the author?
2. If the data is real, describe whether or not Census data reflects the total population of the United States?
3. If the data is real, how might we improve the model?
4. If the data is not real, does the real data follow a different trend than the made up data?