**Activity 5.4.2 Transformations Using** $f(x)=e^{x}$

In the last activity, you determined the effect of parameters on the function $f(x)=d+a∙b^{cx}$. The transformations are the same transformations that you saw in previous units.

* $g(x)+d$ vertically shifts the graph of $g(x)$
* $a∙g(x)$ vertically stretches or compresses the graph of $g(x)$
* $g(cx)$ horizontally stretches or compresses the graph of $g(x)$

One transformation that you did not see in the last activity is $g(x+m)$. From previous units, you learned that $g(x+m)$ horizontally shifts the graph of $g(x)$.

**Investigating** $f(x+m)$

1. Graph $f(x)=e^{x}$ and $g(x)=e^{x+2}$ on the same coordinate system.
2. Sketch the graphs here.



1. For what x-value does $f(x)=1$?
2. For what x-value does $g(x)=1$?
3. How does the graph of $g(x)$ compare to the graph of $f(x)$?
4. Graph $g(x)=e^{x+2}$ and $h(x)=7.3891e^{x}$ on the same coordinate system.
5. Sketch the graphs here.



1. How does the graph of $g(x)$ compare to the graph of $h(x)$?
2. Let’s see why this is happening. Start with the function $g(x)=e^{x+2}$.
	* 1. Use the product rule of exponents to write $e^{x+2}$ as a product.

$$g(x)=e^{x+2}=e^{}∙e^{}$$

* + 1. One of these factors is an exponential function to the power $x$. The other factor is a constant. Approximate the value of the constant to 4 decimal places. Rewrite the function $g(x) $using the approximation of the constant.
		2. *This example shows that the function* $g(x)=e^{x+m}$ *can be written as a* product $g(x)=a∙e^{x}$. Write an expression for $a$ in terms of $m$.

**Which Numbers Can Be Written as a Power of** $e$**?**

1. Sketch the graph of $f(x)=e^{x}$.



1. On the same coordinate system, graph:
	1. $y=5$
	2. $y=\frac{1}{3}$
2. Does the graph of the line $y=5$ intersect the graph of $f(x)=e^{x}$?
3. Does the graph of the line $y=\frac{1}{3}$ intersect the graph of $f(x)=e^{x}$?
4. For what values of $b$ will the graph of the line $y=b$ intersect the graph of $f(x)=e^{x}$?

**Finding x-values**

You now have a set of b-values for which the equation $b=e^{x}$ has a solution.

1. Go back to the example $5=e^{x}$. Rewrite this exponential equation in logarithmic form to find the exact value of $x$.
2. For what value of $x$ is $\frac{1}{3}=e^{x}?$
3. Choose three values for $b$ (not 5 or $\frac{1}{3}$) and for each $b$, find the value of $x$ so that $b=e^{x}$.
4. Now solve the equation without substituting in values. For what value of $x$ does $b=e^{x}$?

**Forms of Exponential Functions**

All exponential functions can be written in multiple forms including $y=a∙b^{t}$ and $y=a∙e^{rt}$. The equations $y=a∙b^{t}$ and $y=a∙e^{rt}$ are equivalent. Let’s figure out why and determine an expression for $r$.

1. Start with $y=a∙b^{t}$. What restrictions are there on $b$, the base of an exponential function?
2. Explain why$ b$ can be written as a power of $e$.
3. Since b can be written as a power of e, this means $b=e^{r}$ for some value r. Substitute $b=e^{r}$ into the equation $y=a∙b^{t}$. Don’t forget the parentheses around $e^{r}$.
4. Explain how this becomes $y=a∙e^{rt}$.

1. Now figure out an expression for $r$ in terms of $b$. Solve the equation $b=e^{r}$ for $r$.
2. Substitute this expression for $r$ in to the equation $y=a∙e^{rt}$. This is another equation equivalent to the initial two equations!

**Compound Interest**

1. Julia is opening a savings account with an initial deposit of $250. The account offers a 3.6% APR compounded quarterly.
	1. Assuming she makes no other deposits or withdrawals, write a function giving the amount in the account after $t$ years.
	2. Rewrite the function from part a with base $e$.
2. Joachim is open a savings account with an initial deposit of $250. The account offers a 3.5% APR compounded continuously. Assuming he makes no other deposits or withdrawals, write a function giving the amount in the account after $t$ years.
3. Compare the functions for Joachim and Julia (function from part b). Just by comparing the functions, can you tell which person is getting a better deal from the bank? Explain.
4. Verify your answer for question 3 by comparing graphs or a table of values. After 10 years, how much money will Julia have in the bank? How much will Joachim have?

**Population Change**

The number of tri-color bats in Connecticut is decreasing at the rate of 49% per year. Your environmental studies class has counted 194 bats in the area.

1. Write an exponential function (base $b$) modeling the population of tri-color bats after $t$ years.
2. What part of the function in part a shows that the population is decreasing?
3. Write an exponential function using base $e$ modeling the population of tri-color bats after $t$ years.
4. What part of the function in 3 above shows that the population is decreasing?
5. In how many years will the tri-color bat population be below 5 bats?