**Activity 5.4.1 Changing Parameters**

In the launch activity, you were given a situation that started with colony of 400 bacteria. The bacteria population doubled every four hours. You saw that the situation was represented by the function $f(t)=400\left(2^{0.25}\right)^{t}$ where $t $is the number of hours elapsed. In this activity, you are going to change the values of parameters in the function $f(t)=d+a\left(b^{c}\right)^{t}$ and determine the effects of the parameters on the graph.

**Basic Situation**

Look at the function $k(t)=400\left(2\right)^{t}$.

Imagine that this function represents the population of a bacteria colony after t hours.

1. Fill in the missing values in the table

|  |  |
| --- | --- |
| $$t$$ | $$k(t)$$ |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 16 |  |

1. Why is it acceptable to use negative values for time?
2. How long does it take for the population of the bacteria colony to double?
3. Graph this basic function $k(t)=400\left(2\right)^{t}$ with the graph of $f(t)=400\left(2^{0.25}\right)^{t}$. Feel free to use technology. How are the graphs similar? How are they different?

**Situation 1 – Parameter** $c$

Look at the function $g(t)=400\left(2^{4}\right)^{t}$.

Imagine that this function represents the population of a bacteria colony after t hours.

1. Fill in the missing values in the table

|  |  |
| --- | --- |
| $$t$$ | $$g(t)$$ |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 16 |  |

1. How long does it take for the population of the bacteria colony to double?
2. How can you tell by looking at the function how long it takes for the bacteria population to double?
3. Graph this basic function $k(t)=400\left(2\right)^{t}$ with the graph of $g(t)=400\left(2^{4}\right)^{t}$. Feel free to use technology. How are the graphs similar? How are they different?

**Generalization for Parameter** $c$

You have now graphed a basic function $k(t)=400\left(2\right)^{t}$ and two transformed functions $f(t)=400\left(2^{0.25}\right)^{t}$ and $g(t)=400\left(2^{4}\right)^{t}$. Note that the only change in these functions is the value of the parameter c in the general function $f(t)=a\left(b^{c}\right)^{t}$. Compare your graphs for the three functions and make a conjecture about the effect of c on the graph of the function $y=a∙b^{t}$.

**More about the Parameter** $c$

In the function $g(t)$, you had this expression $\left(2^{4}\right)^{t}$. If you use the order of operations to simplify the expression inside the parentheses, you get $16^{t}$.

1. Fill in the missing values.

|  |  |  |
| --- | --- | --- |
| $$t$$ | $$\left(2^{4}\right)^{t}=16^{t}$$ | $$2^{4t}$$ |
| -2 |  |  |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 5 |  |  |

1. What do you notice about the last two columns of the table?
2. Why does this happen? (Hint: Think about exponent properties.)

**Situation 2 – Parameter** $d$

The scientist has two containers each with the same type of bacteria colony. One of the containers is small and is already at maximum capacity with only 500 bacteria. Assume that the population in this colony has been at the 500 level for some time and will remain constant. The other container is significantly larger. There are currently 400 bacteria in the larger container and the population doubles every hour.

1. Write a function $h(t)$ that represents the total bacteria population in both colonies after t hours.
2. Fill in the missing values in the table

|  |  |
| --- | --- |
| $$t$$ | $$h(t)$$ |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 16 |  |

1. Graph this basic function $k(t)=400\left(2\right)^{t}$ with the graph of $h(t)$. How are the graphs similar? How are they different?

**Generalization for Parameter** $d$

You have now graphed a basic function $k(t)=400\left(2\right)^{t}$ and a transformed function $h(t)$. Note that the only difference in these functions is the value of the parameter $d $in the general function $f(t)=d+a\left(b\right)^{t}$. Compare your graphs for the two functions and make a conjecture about the effect of $d$ on the graph of the function $y=a∙b^{t}$.

**What if You Consider Something Other than Doubling?**

1. At an initial observation, the colony contains 400 bacteria. The bacteria population triples every hour. Write a function $g(t)$ that represents the population of the bacteria colony after t hours.
2. At an initial observation, the colony contains 400 bacteria. This time the colony has begun shrinking. The bacteria population is cut in half every hour. Write a function $h(t)$ that represents the population of the bacteria colony after t hours.
3. Compare the functions $g(t)$ and $h(t)$ with the basic function $k(t)=400\left(2\right)^{t}$. What parameter has changed?
4. Compare the graphs of the functions $g(t)$ and $h(t)$ with the basic function $k(t)=400\left(2\right)^{t}$.

**Generalization for Parameter \_\_\_\_\_\_**

You have now graphed a basic function $k(t)=400\left(2\right)^{t}$ and two transformed functions $h(t)$ and $g(t)$. Note that the only change in these functions is the value of the parameter \_\_\_\_ in the general function $f(t)=d+a\left(b^{c}\right)^{t}$. Compare your graphs for the three functions and make a conjecture about the effect of \_\_\_\_ on the graph of the function $y=a∙b^{t}$.

**What if You Have a Different Initial Population?**

1. At an initial observation, the colony contains 200 bacteria. The bacteria population doubles every hour. Write a rule for a function $g(t)$ that represents the population of the bacteria colony after t hours.
2. At an initial observation, the colony contains 1000 bacteria. The bacteria population doubles every hour. Write a rule for a function $h(t)$ that represents the population of the bacteria colony after t hours.
3. Compare the functions $g(t)$ and $h(t)$ with the basic function $k(t)=400\left(2\right)^{t}$. What parameter has changed?
4. Compare the graphs of the functions $g(t)$ and $h(t)$ with the basic function $k(t)=400\left(2\right)^{t}$.

**Generalization for Parameter \_\_\_\_\_\_**

You have now graphed a basic function $k(t)=400\left(2\right)^{t}$ and two transformed functions $h(t)$ and $g(t)$. Note that the only change in these functions is the value of the parameter \_\_\_\_ in the general function $f(t)=d+a\left(b^{c}\right)^{t}$. Compare your graphs for the three functions and make a conjecture about the effect of \_\_\_\_ on the graph of the function $y=a∙b^{t}$.

**Summary**

In this activity, you investigated transformations of exponential functions. In each section, you discovered how changing a parameter affects the graph of the function. Now is your chance to pull the information together in one location. For each parameter, $a, b, c, and d$, describe how the parameter affects the graph of $f(t)=d+a\left(b^{c}\right)^{t}$. Be as specific as possible and include sketches to aid your explanation.

$$f(t)=d+a\left(b^{c}\right)^{t}$$

Parameter $a$:

Parameter $b$:

Parameter $c$:

Parameter $d$:

**Practice**

1. Each given expression in an exponential raised to a power. Rewrite each as an exponential expression (the exponential should not be raised to a power).
	1. $\left(3^{2t}\right)^{5}$
	2. $\left(2^{3}\right)^{4t}$
	3. $\left(1.8^{4}\right)^{0.125x}$
2. Fill in the missing exponential expression.
	1. $4^{18x}=\left(\\_\\_\\_\\_\right)^{3x}$
	2. $5^{7x}=\left(\\_\\_\\_\\_\right)^{2}$
	3. $7^{4x-8}=\left(\\_\\_\\_\\_\right)^{4}$
3. Sketch the graph of $f(t)=3^{t}$.



1. Given the function $g(t)=4∙\left(3^{2}\right)^{t}$ which is a transformation of $f(t)$,
2. describe how the 4 affects the graph of $f(t)$, and
3. describe how the 2 affects the graph of $f(t)$.

Sketch the graph of $g(t)$ on the same coordinate system as $f(t)$.

1. Sketch the graph of $f(t)=\left(\frac{1}{2}\right)^{t}$.

 

1. Given the function $g(t)=2∙\left(\frac{1}{2}\right)^{5t}-3$ which is a transformation of $f(t)$,
2. describe how the 2 affects the graph of $f(t)$,
3. describe how the 3 affects the graph of $f(t)$, and
4. describe how the 5 affects the graph of $f(t)$.
5. Sketch the graph of $g(t)$ on the same coordinate system as $f(t)$.
6. John invested $1000 at 4% interest compounded monthly, Jose invested $1100 at 3.8% interest compounded continuously and Mary invested $1100 at 3.9% compounded annually.
	1. Determine the 3 functions that can be used to determine the amount teach person has after t years.
	2. Evaluate each function at 10 and interpret.
	3. Graph and use the table feature to study the behavior of these 3 functions for 40 years and write down at least 3 observations you have made. Be sure to include which person has the largest amount after 40 years.
	4. Would you expect the same person to have the largest amount at 50 years? Explain. Check your table. Are you correct?
	5. Check again at 60 years. Are you still correct?