**Unit 4: Investigation 3 (3 Days)**

**Proving Similar Triangles**

**Common Core State Standards**

* G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
* G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Overview**

In this investigation students will explore “shortcuts” for proving triangles are similar. Through discovery, students develop three conjectures, the AA Similarity Conjecture, SAS Similarity Conjecture, and SSS Similarity Conjecture. Each of these conjectures are then proven and used as theorems in subsequent problems. Using these theorems, students are then able to prove other theorems such as, all equilateral triangles are similar.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Prove AA Similarity, SAS Similarity, and SSS Similarity theorems.
* Use similarity transformations and similarity theorems to prove two or more triangles similar.
* Prove that all equilateral triangles are similar.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 4.3.1** asks students to prove two triangles are similar
* **Exit Slip 4.3.2** asks students to prove two triangles similar and then to use the corresponding parts property.
* **Journal Entry** asks students to explain why SSA is not criteria for proving two triangles are similar

**Launch Notes**

You may begin this investigation by building off students’ prior knowledge of the criteria for proving two figures similar. At this point students will find the work of proving triangles similar by comparing all pairs corresponding parts tedious. This drives the question as to whether two triangles are similar with limited information. In addition, students have explored the minimum criteria or the three conditions that suffice to show two triangles are congruent and therefore may already be thinking that there are minimum criteria to prove triangles are similar. **Activity 4.2.1** explores the minimum criterion needed for two triangles to be similar.

**Teaching Strategies**

In **Activity 4.3.1** **Triangle Similarity Conjectures** students are put into pairs and each are asked to draw individual triangles using the given equal measurements. In the first case, students question if they know that three angles of one triangle are congruent to three angles of another triangle then will the lengths of the pairs of corresponding sides be in proportion and thus the triangles similar. In pairs, students each create their own triangle with the interior angle measures: $35°,60°, $and $85°$. Each student will now have two different size triangles but their corresponding angles are congruent. Students will then measure the lengths of the sides and calculate the ratio of the lengths of the corresponding sides for their pairs of triangles. Due to drawing errors and approximations in measuring, the ratios that students find should be close but might not be equal. Students should be comfortable with this idea and understand the imprecision of their measurements. It is through this hands-on action that students should realize and use the Third Angles Theorem: if two angles of one triangle congruent to two angles of another triangle, then the third pair of angles are also congruent. This is why we have an AA Similarity Conjecture and not an AAA Similarity Conjecture.

**Group Activity**

The advantage to using **Activity 4.3.1** with pairs of two and not groups of three or more is that when students are required to compare the ratios of their pairs of corresponding side lengths they will save time by only comparing with one student rather than two. Each pair may then present their findings to the class as a whole and learn that they all have triangles that are similar.

Have pairs of students report their findings to the class as a whole. A whole class discussion should take place to establish that if two angles of one triangle are congruent to two angles of another triangle then the lengths of the corresponding sides will be in proportion and thus the triangles are similar. Have students determine an appropriate name for this conjecture, AA Similarity Conjecture.

**Note:** For this rest of this activity you will need to have sets of pre-cut straws with the following lengths: 2 in, 3 in (two copies), 4 in, 4.5 in, and 6 in. These should be prepared in advance and placed in zip-lock bags. If you have students work in pairs you will need enough bags for each pair.

Next students will explore SSS and SAS similarity. Six pre-cut straws are used to create two different size triangles of which the lengths of the corresponding sides are in proportion. Students then measure the angles formed by the sides to learn that if the lengths of the corresponding sides are in proportion then the corresponding angles are congruent. Through a whole class discussion, students establish the SSS Similarity Conjecture.

For SAS the students use four straws and a given angle measure. They will construct the third side and much as they did for AA Similarity, they will then measure the parts of the triangles. Each group will then compare the measures of their corresponding angle and write ratios of corresponding side lengths. From this students will discover the SAS Similarity Conjecture.

You may give students **Exit Slip 4.3.1** before you give Activities 4.3.2 and 4.3.3 provided that you allow students to use the conjectures generated in Activity 4.3.1 in their proofs.

**Journal Entry**

Explain why there is no SSA Similarity Theorem. Look for students to recognize that when the angle is not included between the sides, there may be more than one triangle that fits the SSA criterion.

In **Activity 4.3.2 The AA and SAS Similarity Theorem**, students prove their AA Similarity Conjecture and SAS Similarity Conjecture as theorems. These proofs involve transformations. Students fill in the blanks to complete the proof.

In **Activity 4.3.3** **The SSS Similarity Theorem** students prove their SSS Similarity Conjecture as a theorem. This proof involves constructing a triangle congruent to one of the triangles that can be proved similar to the other one. More of the work is left to the students than in the previous activity.

**Differentiated Instruction (For Learners Needing More Help)**

Some students may need more guidance completing the proof in Activity 4.3.3

**Activity 4.3.4 Using Similarity Theorems** requires students to use AA Similarity Theorem, SAS Similarity Theorem, and SSS Similarity Theorem in various proofs. Proofs may be written in two column, paragraph, and flow chart formats. Included in this activity are questions for which students use Corresponding Sides of Similar Triangles are in Proportion (CSSTP) or Corresponding Angles of Similar Triangles are Congruent (CASTC). One way to differentiate this lesson/task is to provide students with some of the statements or reasons in the proof.

Following Activity 4.3.4 you may give students **Exit Slip 4.3.2.**

**Activity 4.3.5 Similarity in Equilateral Triangles** may be assigned at any time after **Activity 4.3.2.** In this activity students discover and then prove that all equilateral triangles are similar.

**Differentiated Instruction (Enrichment):** We have proved that all circles are similar and all equilateral triangles are similar. Have students discover what other figures, in particular regular polygons, have this property.

**Closure Notes**

Ask students to summarize the three ways they have learned to prove that two triangles are similar. Ask them to speculate about whether these combinations of congruent parts will also guarantee that triangles are congruent: SS or SSA.

**Abbreviations**

Corresponding Angles of Similar Triangles are Congruent (CASTC)

Corresponding Sides of Similar Triangles are in Proportion (CSSTP)

**Theorems**

**AA Similarity Theorem**: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

**SAS Similarity Theorem:** If two sides of one triangle are proportional to two sides of another triangle and the angles included by these sides are congruent, then the triangles are similar.

**SSS Similarity Theorem:** If three sides of one triangle are proportional to three sides of another triangle, then the two triangles are similar.

**Equilateral Triangle Similarity Theorem:** All equilateral triangles are similar.

**Resources and Materials**

Activities:

 Activity 4.3.1 Triangle Similarity Conjectures

 Activity 4.3.2 The AA and SAS Similarity Theorems

 Activity 4.3.3 The SAS Similarity Theorem

 Activity 4.3.4 Using Similarity Theorems

 Activity 4.3.5 Similarity in Equilateral Triangles

Materials: Rulers, Protractors, Straws (pre-cut, see note above), zip-lock bags and GeoGebra