**Unit 4: Investigation 1 (2 Days)**

**Dilations**

**Common Core State Standards**

* G-SRT.A.1.A A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
* G-SRT.A.1.A The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

**Overview**

In this investigation students will study dilation as a transformation in the plane. To begin, students will discover the properties of dilations by dilating geometric shapes without the use of technology. Students will then use technology to further investigate these properties and develop a dilation postulate that they will use in proving triangles similar.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Understand that dilations map lines not passing through the center of dilation to a parallel line and lines passing through the center of dilation remain unchanged.
* Construct dilations of figures using a compass and straightedge when the center of dilation is on the figure, in the interior of the figure, or in the exterior of the figure.
* Identify the scale factors of images that have been dilated.
* Draw the image of a given image that has undergone a dilation.
* Determine if two given figures are the result of a dilation and if so, determine the center of dilation.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 4.1.1** asks students to determine if figure one figure is the image of the other under a dilation.
* **Exit Slip 4.1.2** asks students to determine the center of dilation given a pre-image and image.
* **Journal Entry** asks students to describe the location of the images of various lines under dilation.

**Launch Notes**

Students may have encounterd dilations in middle school; in fact, as early as 7th grade; under the mathematics Common Core State Standards, students begin working with scale drawings (7.G.A.1). In 8th grade, standards (8.G.A.3 & 8.G.A.4) require students to describe the effect of dilations in the Cartesian coordinate plane as well as identify figures that are similar. It is now in high school where students will develop a formal understanding of the properties of a dilation based on prior knowledge and discovery based investigations.

Begin this investigation with real life examples of various dilations that are familiar to your students. Some examples are a dilated pupil, Matchbox cars (scale is given on the bottom of car), and building models. This will help facilitate a discussion about dilations and bring about their prior knowledge. An engaging PBS video located at <http://www.pbslearningmedia.org/resource/mgbh.math.rp.humantree8/human-tree-dilations/> can help motivate students for the lesson. The entire video runs for 2 minutes and 20 seconds, but you may only want to show the first minute or so to get the basic idea of dilation.

**Teaching Strategies**

 Use **Activity 4.1.1** **Properties of Dilations** to help students discover the properties of a dilation by hand. Each student is given a piece of paper that contains a polygon and a point (center of dilation) in the exterior of the figure. Explain to students that they are to dilate the figure, about the center of dilation *P*, by a scale factor of 2. Have each student draw rays emanating from the center of dilation through each vertex of the figure. Then using a compass, have students measure the distance from the center of the dilation to a vertex. Now place the pointer of the compass on the vertex and measure that same distance but further along the ray. The student will now have a point that is twice as far from the center of dilation as the pre-image vertex. Have them continue this until all vertices of the image are created. Next, have the students discuss things they notice about the parts of the pre-image and the parts of the image. They should use a ruler and protractor to compare measurements between the pre-image and image.

Repeat this process using a new figure, but change the scale factor to a value that is between 0 and 1. The directions in the activity use a scale factor of 1/2. Students can find the midpoint of a segment using the compass and straightedge construction introduced in Activity 2.7.6 or the Midpoint Formula for the coordinate plane. Keep the center of dilation in the exterior of the closed figure.

After students construct the image of the dilated figure have them measure the corresponding parts and compare the results with the first dilation. At this point students have constructed two dilations and have seen that the ratio of the corresponding side lengths from the image to the pre-image is equal to the scale factor and that the measures of the corresponding angles are equal.

Now have students focus on the position of the image in the plane as a result of the dilation. Students may need some time to think about this question. Let them struggle at first but through group discourse they should verbalize that some corresponding sides of the polygons from the pre-image and image are parallel.

Following **Activity 4.1.1** you may give **Exit Slip 4.1.1**.

**Group Activity:** Have students work on **Activity 4.1.1** in heterogeneous groups. They can check each other’s work and resolve any discrepancies. Groups should be designed to facilitate strong student-to-student discourse.

**Differentiated Instruction (For Learners Needing More Help)**

The figures produced in Activity 4.1.1 may be visually confusing to some students. They may benefit from using colored pencils. Choose one color for the pre-image, one from the image, and another from the lines radiating from the center of dilation.

Use **Activity 4.1.2** **Dilation GeoGebra Exploration** to continue to investigate the properties of a dilation through the use of dynamic geometry software. In this investigation students explore what happens when the center of dilation is in the interior of the figure as well on a vertex of the pre-image. In this investigation students should be encouraged to grab points on the pre-image and move them around at the same time they are noticing changes in the image. Have students discuss their findings and compare them to the results of **Activity 4.1.1.**

Following **Activity 4.1.2** you may give **Exit Slip 4.1.2**.

**Journal Entry:** Under a dilation, when does a line map onto itself? When does it map onto another line? When it maps onto a second line what is the relationship between the two pre-image line and the image line? Look for students to explain the distinct behavior of lines that pass through the center of dilation and those that do not.

**Differentiated Instruction (Enrichment):** Have students experiment with dilations that have negative scale factors (such as –1, –2, and $-\frac{1}{2}$) and make conjectures based on their observations.

**Closure Notes**

Now that students have thoroughly explored the properties of a dilation by hand and through the use of technology they should be able to develop a dilation postulate. Have individuals state properties they have observed and make a list on the board. The resulting postulate should contain the following ideas:

(1) Dilations preserve angle measure. (2) They map parallel lines onto parallel lines and midpoints onto midpoints. (3) The length of the image of a segment is the length of the segment times the scale factor. (4) If a line passes through the center of dilation it is mapped onto itself. (5) If a line *l* does not pass through the center of dilation it is mapped onto a line *l’* parallel to *l.*

**Vocabulary**

center of dilation

dilation

**Postulate**

**Dilation Postulate:** Dilations preserve angle measure and betweenness. They map parallel lines onto parallel lines and midpoints onto midpoints. The length of the image of a segment is the length of the segment times the scale factor. If a line passes through the center of dilation it is mapped onto itself. If a line *l* does not pass through the center of dilation it is mapped onto a line *l’* parallel to *l.*

**Resources and Materials**

Compass, ruler, and protractor for Activity 4.1.1

GeoGebra for Activity 4.1.2

Rulers for the exit slips

Activity 4.1.1 Properties of Dilation

Activity 4.1.2 Dilation Geogebra Exploration

Exit Slip 4.1.1

Exit Slip 4.1.2

National Museum of Mathematics creates a "Human Tree" using dilations:

<http://www.pbslearningmedia.org/resource/mgbh.math.rp.humantree8/human-tree-dilations/>