**Unit 3: Polygons**

**(4-5 Weeks)**

**UNIT PLAN**

An in depth study of the properties of polygons provides students with the opportunity to construct a sequence of theorems based on the postulates and theorems introduced in unit 2. During this unit as they learn new facts about polygons, they should also hone their ability to understand and write proofs. Many of the properties studied will first be introduced through experimentation as students formulate conjectures. The teacher should continually emphasize that supporting a conjecture by evidence (inductive reasoning) does not constitute a proof (deductive reasoning).

In **Investigation 1** students use transformations to prove that the sum of the interior angles of a triangle is 180°. They extend this result to quadrilaterals and then show that sum of the interior angles of a convex polygon with *n* sides is (*n* – 2)180°.

From the Triangle Sum Theorem it follows that an exterior angle of a triangle is greater than either non-adjacent interior angle. This is the first of several theorems about inequalities in triangles featured in **Investigation 2**, culminating in the Triangle Inequality.

In **Investigation 3** the property of exterior angles in triangles is used to prove the converse of the Parallel Lines Corresponding Angles Theorem. It is an opportunity to solidify understanding of the relationship between a conditional statement and its converse. In addition, the converse is used to justify the compass and straightedge construction of a line through a given point parallel to a given line.

Regular polygons are studied in **Investigation 4**. Students learn how to use compass and straight edge construct three regular polygons: equilateral triangles, squares, and hexagons given either the length of a side or the radius of a circumscribed circle.

The properties of special quadrilaterals are studied in **Investigations 5 and 6**. In some cases a theorem is proved twice, first by synthetic methods and then by coordinate methods so that students may compare and contrast these methods of proof. Theorems and midsegments of triangles and trapezoids and the concurrency of the medians in a triangle are proved by coordinate methods in **Investigation 6**.

The unit culminates in an introduction to tessellations in **Investigation 7.** We revisit the properties of regular polygons to show that there are exactly three regular tessellations. The Triangle Sum and Quadrilateral Sum theorems together with an understanding of transformations are used to show that any triangle and any quadrilateral will tile the plane.

**Essential Questions**

* How are polygons classified?
* How can we prove properties of polygons?
* How are theorems and their converses related to each other?
* How can regular polygons be constructed?
* How does coordinate proof differ from synthetic proof?
* Which polygons will tile the plane?

**Enduring Understandings**

Theorems about polygons are related to each other in that all are proved based on postulates and previously proved theorems.

**Unit Contents**

Investigation 1 Sums of Interior Angles of Polygons (3 days)

Investigation 2, Inequalities in Triangles (2 days)

Investigation 3 Parallel and Perpendicular Lines (2 days)

Mid Unit Test (1 day)

Investigation 4 Regular Polygons (3 days)

Investigation 5 Properties of Quadrilaterals (3 days)

Investigation 6 Polygons with Coordinates (3 days)

Investigation 7 Introduction to Tessellations (2 days)

Performance Task “Designing Shape Blocks” (1 day)

End of Unit Test (2 days including review)

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. **Construct viable arguments and critique the reasoning of others.**

4. Model with mathematics.

5. **Use appropriate tools strategically.**

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Standards Overview**

* Prove geometric theorems
* Make geometric constructions
* Use coordinates to prove simple geometric theorems algebraically

**Standards**

G-CO.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;**the medians of a triangle meet at a point.*

G-CO.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

**Vocabulary**

Base (of Trapezoid)

“Bisect each other”

Centroid (of Triangle)

Circumscribed circle

Concurrent lines

Converse

Convex polygon

Coordinate proof

Diagonal

Exterior angle of a polygon

Heptagon

Hexagon

Inscribed polygon

Interior angle of a polygon

Isosceles Trapezoid

Kite

Leg (of Trapezoid)

Median (of Triangle)

Midsegment (of Trapezoid, triangle)

Necessary condition

Non-adjacent interior angle of a triangle

Non-convex polygon

Octagon

Parallelogram

Quadrilateral

Rectangle

Regular tessellation

Rhombus

Square

Sufficient condition

Synthetic proof

Tessellation

Tiling

Transitive Property for >

Trapezoid

Trichotomy principle

**Theorems**

**(3-1-1) Triangle Sum Theorem:** In any triangle the sum of the interior angles is 180°.

**(3-1-2) Quadrilateral Sum Theorem:** In any convex quadrilateral the sum of the interior angles is 360°.

**(3-1-3) Polygon Sum Theorem:** In any convex polygon with *n* sides, the sum of the interior angles is 180°(*n* – 2).

**(3-1-4) Third Angles Theorem:** If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are also congruent.

**(3-1-5) AAS Congruence Theorem:** It two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

**(3-2-1) Exterior Angle of a Triangle Theorem:** The measure of an exterior angle of any triangle is equal the sum of the measures of the two non-adjacent interior angles.

**(3-2-2) Exterior Angle of a Triangle Corollary:** The measure of an exterior angle of any triangle is greater than the measure of either non-adjacent interior angle.\*

**(3-2-3) Angle Opposite Longer Side Theorem:** If two sides of a triangle are not equal, the angle opposite the longer side is greater than the angle opposite the smaller side.

**(3-2-4) Side Opposite Larger Angle Theorem:** If two angle of a triangle are not equal, the side opposite the larger angle is longer than the side opposite the smaller angle.

**(3-2-5) Triangle Inequality Theorem:** The sum of the lengths of any two sides of a triangle is greater than the third side.

**(3-3-1) Corresponding Angles Congruent 🡪 Parallel Lines (Parallel Lines, Corresponding Angles Converse):** If two lines are cut by a transversal so that a pair of corresponding angles are congruent, then the lines are parallel.

**(3-3-2) Alternate Interior Angles Congruent 🡪 Parallel lines (Parallel Lines, Alternate Interior Angles Converse):** If two lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the lines are parallel.

**(3-3-3) Perpendicular and Parallel Lines Theorem:** If two lines are perpendicular to the same line, then they are parallel.

**(3-4-1) Exterior Angles of Polygon Theorem:** The sum of the exterior angles of a [convex] quadrilateral is 360°.

**(3-4-2) Angles in Regular Polygon Theorem:** In a regular polygon the measure of each interior angle is $\frac{\left(n-2\right)180°}{n}$ and the measure of each exterior angle is $\frac{360°}{n}$.

**(3-5-1) Parallelogram Opposite Sides Theorem:** If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent.

**(3-5-2) Parallelogram Opposite Sides Converse:** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**(3-5-3) Parallelogram Diagonals Theorem:** If a quadrilateral is a parallelogram, then the diagonals bisect each other.

**(3-5-4) Parallelogram Diagonals Converse:** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

**(3-5-5) Rectangle Diagonals Theorem:** If a parallelogram is a rectangle, then the diagonals are congruent.

**(3-5-6) Rectangle Diagonals Converse:** If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

**(3-5-7) One Pair Congruent and Parallel Theorem:** If one pair of sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.

**(3-5-8) Parallelogram Opposite Angles Theorem:** If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent.

**(3-5-9) Parallelogram Opposite Angles Converse:** If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**(3- 5-10) Rhombus Diagonals Theorem:** If a parallelogram is a rhombus, then the diagonals are perpendicular.

**(3-5-11) Rhombus Diagonals Converse:** If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

**(3-6-1) Trapezoid Midsegment Theorem:** The segment joining the two legs of a trapezoid is parallel to the bases and equal in length to the average of the lengths of the two bases.

**(3-6-2) Triangle Midsegment Theorem:** The segment joining two sides of a triangle is parallel to the third side and equal in length to half the third side.

**(3-6-3) Centroid Theorem:** The three medians of a triangle are concurrent at the centroid.

**Constructions**

 **(3-3-1) Line through a given point parallel to a given line**

**(3-4-1) Equilateral Triangle Constructions (side, radius of circumscribed circle)**

**(3-4-2) Square Constructions (side, radius of circumscribed circle)**

**(3-4-3) Regular Hexagon Constructions (side, radius of circumscribed circle)**

**Assessment Strategies**

**Performance Task**

Students are given the task of designing a set of blocks that can be used in Pre-K and Kindergarten classes to teach children about shapes. The constraints they are given are the minimum number of required shapes, the types of shapes (special triangles and quadrilaterals), the maximum and minimum area of each block to be cut from a 12 X 12 unit slab of wood, and the maximum amount of wasted material allowed. They are to present their solution by indicating the vertices of the shapes in a coordinate plane and using coordinate geometry to categorize the shapes and determine their areas.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* Mid-unit test
* End-of-unit test