**Activity 3.6.5 Midsegments**



Open the file ctcoregeomACT365.ggb. The vertices of quadrilateral *ABCD* lie on two parallel lines as shown.

Move point *C* to demonstrate that DC is always parallel to $\overbar{AB}$.

Move point *D* to show that *D* always lies on the line through *C* parallel to $\overbar{AB}$. (However, don’t move *D* beyond *C*—we’ll do that later.)

1. Classify *ABCD* as a special quadrilateral. *ABCD* must be a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. Why?

2. The lengths of segment $\overbar{ED}$, $\overbar{EA}$, $\overbar{FC}$, and $\overbar{FB}$ are shown. As you move any of the vertices of the quadrilateral these quantities may change, but what always stays the same?

3. Therefore *E* is the \_\_\_\_\_\_\_\_\_\_ of $\overbar{AD}$ and *F* is the \_\_\_\_\_\_\_\_\_\_\_ of$\overbar{ BC}$.

4. In a trapezoid the two parallel sides are called **bases** and the other two sides are called **legs**. In *ABCD* the bases are \_\_\_\_\_ and \_\_\_\_. The legs are \_\_\_\_\_\_ and \_\_\_\_\_\_.

5. The **midsegment** of a trapezoid is the segment joining the midpoints of the legs. Name the midsegment of *ABCD*: \_\_\_\_\_\_\_\_\_

6. Compare the length of the midsegment of *ABCD* with the sum of the two bases. What do you notice? Does this relationship still hold when you move the vertices?

7. Find the slope of the midsegment of *ABCD* and compare it to the slopes of the bases. What do you notice? Does this relationship still hold when you move the vertices?

8. Based on your observations make a conjecture:

9. Now move *D* so that it coincides with *C*. What type of figure do you have now?

10. Continuing moving *D* so that it is on the other side of *C* from where it started. What happens? Is *ABCD* still a quadrilateral? Justify your answer.

Here are two theorems based on what we have observed:

**Trapezoid Midsegment Theorem:** The segment joining the two midpoints of the legs of a trapezoid is parallel to the bases and equal in length to the average of the lengths of the two bases.

**Triangle Midsegment Theorem:** The segment joining the midpoints of two sides of a triangle is parallel to the third side and equal in length to half the third side.

11-12. Prove the Trapezoid Midsegment Theorem. Recall that a trapezoid may be represented in the coordinate plane by *P*(0,0), *Q*(*a*, 0), *R*(*d*, *c*) and *S*(*b*, *c*). Let *T* be the midpoint of $\overbar{PS}$ and *U* the midpoint of $\overbar{QR}$.

11. Start with the specific case where *a* = 10, *b* = 4, *c* = 6, and *d* = 8, that is the coordinates are *P*(0,0), *Q*(10, 0), *R*(8,6) and *S*(4,6).

a. Find coordinates of *T*(\_\_\_,\_\_\_\_) and *U*(\_\_\_\_\_, \_\_\_\_\_)

b. Find these lengths: *PQ* = \_\_\_\_\_ *SR* = \_\_\_\_\_ *TU* =\_\_\_\_\_\_

c. Find the slopes of $\overbar{PQ}$, $\overbar{SR}$, and $\overbar{TU}:$



d. Now complete the proof.

12. Now prove the theorem in the general case. To make the calculations simpler we can double each variable coordinate so we have *P*(0,0), *Q*(2*a*, 0), *R*(2*d*, 2*c*) and *S*(2*b*, 2*c*).



13. Prove the Triangle Midsegment Theorem. Use the coordinates shown in the figure at the right.