**Activity 3.5.6a Rectangles and Rhombuses**

A rectangle is defined as an equiangular quadrilateral and a rhombus is defined as an equilateral quadrilateral. In this investigation, you will prove necessary and sufficient conditions for rectangles and rhombuses.

**Review of Necessary Conditions of Rectangles and Rhombuses**

**1.** If a quadrilateral is a rectangle, then the diagonals are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**2.** If a quadrilateral is a rhombus, then the diagonals are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Before we prove these conjectures, we need to prove that all rectangles are parallelograms and all rhombuses are parallelograms.

**3. Prove that *All Rectangles are Parallelograms***

Given: *ABCD* is a rectangle.

Prove: *ABCD* is a parallelogram.

Since *ABCD* is a rectangle, $m∠A=m∠B=m∠C=m∠D$

This means that pairs of opposite angles are congruent: , $m∠A= \\_\\_\\_\\_\\_\\_$and $m∠B=\\_\\_\\_\\_\\_\\_$

In Activity 3.5.5 we proved that If a quadrilateral has two pairs of opposite angles that are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then the quadrilateral is a parallelogram. Since *ABCD* has two pairs of opposite angles congruent, it must be a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.**

**4. Prove that *All Rhombuses are Parallelograms.***

Given: *ABCD* is a rhombus.

Prove: *ABCD* is a parallelogram.

Since *ABCD* is a rhombus, *AB = BC = CD = DA.*

This means that pairs of opposite sides are congruent:; *AB = \_\_\_\_* and *BC* = \_\_\_\_\_\_.

In Activity 3.5.5 we proved that If a quadrilateral has two pairs of opposite sides that are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then the quadrilateral is a parallelogram. Since *ABCD* has two pairs of opposite sides congruent, it must be a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.**

5. Prove **Rectangle Diagonals Theorem:** If a parallelogram is a rectangle, then the diagonals are congruent.

Given: *ABCD* is a rectangle.

Prove: $\overbar{AC}≅\overbar{BD}$

First, develop a plan for your proof by thinking backwards.

**a)** Name two triangles that you can prove are congruent that have the diagonals as corresponding parts. Hint: They may be overlapping triangles.

**b)** What three parts of those triangles can you prove are congruent?

Fill in the blanks in the proof below.

Since *ABCD* is a rectangle, opposite sides are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. Therefore, \_\_\_\_\_\_\_\_$≅$\_\_\_\_\_\_\_\_\_. It is also true that \_\_\_\_\_\_\_\_$≅$\_\_\_\_\_\_\_\_\_ because it is a shared side. By definition of rectangle, $∠$\_\_\_\_\_\_\_\_\_$≅∠$\_\_\_\_\_\_\_\_\_. By SAS, $Δ$\_\_\_\_\_\_\_\_\_\_\_$≅ Δ$\_\_\_\_\_\_\_\_\_\_\_.

$\overbar{AC}≅\overbar{BD}$ by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

6. Prove **Rhombus Diagonals Theorem:** If a parallelogram is a rhombus, then the diagonals are perpendicular.

Given: *RBMH* is a rhombus.

Prove: $\overbar{RM}⊥\overbar{BH}$

Since *ABCD* is a parallelogram and the diagonals of a parallelogram \_\_\_\_\_\_\_\_\_\_\_ each other, *OH* = \_\_\_\_.

Also since the sides of a rhombus are congruent *HR* = *RB*.

*OR* = *OR* by the reflexive property.

Therefore ∆*ROH*$ ≅$ ∆*ROB* by the \_\_\_\_\_\_\_\_ Congruence Theorem.

$∠$\_\_\_\_\_\_\_\_\_$≅∠$\_\_\_\_\_\_\_\_\_ because corresponding parts of congruent triangles are congruent.

By the Linear Pair postulate: m$∠$\_\_\_\_\_\_\_\_\_$ +$ m$∠$\_\_\_\_\_\_\_\_\_ = 180°.

Explain why we can now conclude that $\overbar{RM}$ and $\overbar{BH}$ are perpendicular:

**7. Prove Rectangle Diagonals Converse:** If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Given: *RTCE* is a parallelogram.

 $\overbar{RC}≅\overbar{TE}$

Prove: *RTCE* is a rectangle.

Since *RTCE* is a parallelogram, opposite sides are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, so $\overbar{RE}≅\overbar{TC}$. $\overbar{EC}≅\overbar{EC}$ because \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. Since it is also given than $\\_\\_\\_\\_\\_\\_\\_\\_≅\\_\\_\\_\\_\\_\\_\\_\\_$, by \_\_\_\_\_\_\_\_\_\_ triangle congruence, it follows that $Δ$\_\_\_\_\_\_\_\_\_\_\_$≅ Δ$\_\_\_\_\_\_\_\_\_\_\_. By CPCTC, $∠REC≅\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$. Since it has been proven that opposite angles of a parallelogram are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then all four angles are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. Then by definition of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**8. Prove Rhombus Diagonals Converse:** If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Given: *RBMH* is a parallelogram.

 $\overbar{RM}⊥\overbar{BH}$

Prove: *RBMH* is a rhombus.

Plan: Use the parallelogram diagonals theorem to find congruent segments. Explain why all four small triangles are congruent in the figure. Then, explain how that proves the figure is a rhombus.

**Write the proof.**