**Activity 3.5.6a Rectangles and Rhombuses**

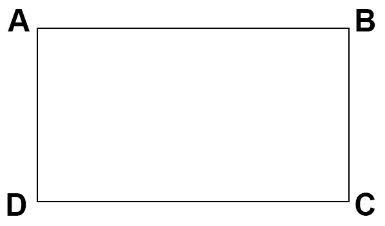
A rectangle is defined as an equiangular quadrilateral and a rhombus is defined as an equilateral quadrilateral. In this investigation, you will prove necessary and sufficient conditions for rectangles and rhombuses.

**Review of Necessary Conditions of Rectangles and Rhombuses**

**1.** If a quadrilateral is a rectangle, then the diagonals are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**2.** If a quadrilateral is a rhombus, then the diagonals are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Before we prove these conjectures, we need to prove that all rectangles are parallelograms and all rhombuses are parallelograms.

**3. Prove that *All Rectangles are Parallelograms***

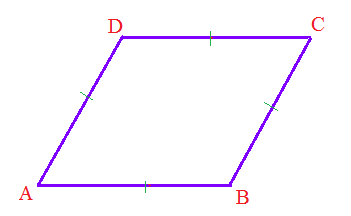
Given: *ABCD* is a rectangle.

Prove: *ABCD* is a parallelogram.

Since *ABCD* is a rectangle,

This means that pairs of opposite angles are congruent: , and

In Activity 3.5.5 we proved that If a quadrilateral has two pairs of opposite angles that are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then the quadrilateral is a parallelogram. Since *ABCD* has two pairs of opposite angles congruent, it must be a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.**

**4. Prove that *All Rhombuses are Parallelograms.***

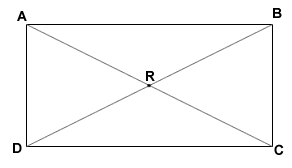
Given: *ABCD* is a rhombus.

Prove: *ABCD* is a parallelogram.

Since *ABCD* is a rhombus, *AB = BC = CD = DA.*

This means that pairs of opposite sides are congruent:; *AB = \_\_\_\_* and *BC* = \_\_\_\_\_\_.

In Activity 3.5.5 we proved that If a quadrilateral has two pairs of opposite sides that are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then the quadrilateral is a parallelogram. Since *ABCD* has two pairs of opposite sides congruent, it must be a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.**

5. Prove **Rectangle Diagonals Theorem:** If a parallelogram is a rectangle, then the diagonals are congruent.

Given: *ABCD* is a rectangle.

Prove:

First, develop a plan for your proof by thinking backwards.

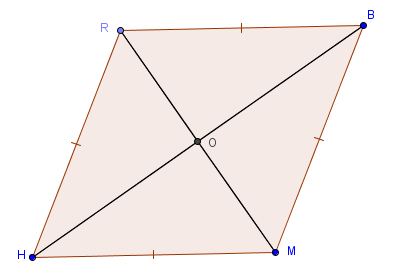
**a)** Name two triangles that you can prove are congruent that have the diagonals as corresponding parts. Hint: They may be overlapping triangles.

**b)** What three parts of those triangles can you prove are congruent?

Fill in the blanks in the proof below.

Since *ABCD* is a rectangle, opposite sides are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. Therefore, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. It is also true that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ because it is a shared side. By definition of rectangle, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. By SAS, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

6. Prove **Rhombus Diagonals Theorem:** If a parallelogram is a rhombus, then the diagonals are perpendicular.

Given: *RBMH* is a rhombus.

Prove:

Since *ABCD* is a parallelogram and the diagonals of a parallelogram \_\_\_\_\_\_\_\_\_\_\_ each other, *OH* = \_\_\_\_.

Also since the sides of a rhombus are congruent *HR* = *RB*.

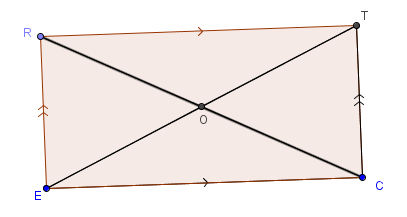
*OR* = *OR* by the reflexive property.

Therefore ∆*ROH* ∆*ROB* by the \_\_\_\_\_\_\_\_ Congruence Theorem.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ because corresponding parts of congruent triangles are congruent.

By the Linear Pair postulate: m\_\_\_\_\_\_\_\_\_ m\_\_\_\_\_\_\_\_\_ = 180°.

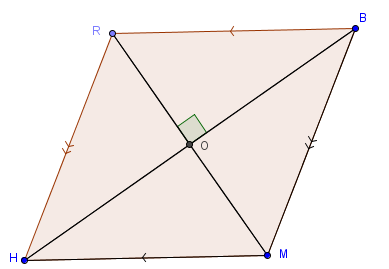
Explain why we can now conclude that and are perpendicular:

**7. Prove Rectangle Diagonals Converse:** If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Given: *RTCE* is a parallelogram.

Prove: *RTCE* is a rectangle.

Since *RTCE* is a parallelogram, opposite sides are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, so . because \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. Since it is also given than , by \_\_\_\_\_\_\_\_\_\_ triangle congruence, it follows that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. By CPCTC, . Since it has been proven that opposite angles of a parallelogram are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then all four angles are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. Then by definition of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**8. Prove Rhombus Diagonals Converse:** If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Given: *RBMH* is a parallelogram.

Prove: *RBMH* is a rhombus.

Plan: Use the parallelogram diagonals theorem to find congruent segments. Explain why all four small triangles are congruent in the figure. Then, explain how that proves the figure is a rhombus.

**Write the proof.**