**Activity 3.5.5 Sufficient Conditions for Parallelograms**

We know that if a quadrilateral has two pairs of parallel sides, then it is a parallelogram by definition. In this activity, you will explore what conditions are sufficient to prove that a quadrilateral has to be a parallelogram, without already knowing that both pairs of opposite sides are parallel.

**1.** Have each person in your group take one piece of linguine and break it so that you have two pairs of equal segments. Form a quadrilateral with your pieces. Depending on how you arrange the pieces you might get a kite, but you should be able to get a parallelogram. Was everyone in your group able to do that?

Now fill in the blanks.

**Parallelogram Opposite Sides Converse:**

If the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_sides of a quadrilateral are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then the quadrilateral is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**2.** Have each person in your group take another piece of linguine and break into two unequal pieces. Let the pieces intersect at their midpoints but not be perpendicular. Either use your pencil or pieces of linguine to outline a quadrilateral around the diagonals. Did everyone in your group successfully make a parallelogram? Now, fill in the blanks.

**Parallelogram Diagonals Converse:**

If the diagonals of a quadrilateral \_\_\_\_\_\_\_\_\_\_\_\_\_\_ each other, then the quadrilateral is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**3.** What if you only know that one pair of opposite sides of a quadrilateral are congruent and parallel, but knew nothing about the other pair of sides? Is that sufficient evidence that the quadrilateral must be a parallelogram? Have each person in your group take a piece of linguine and break off two pieces of equal length. Place them somewhere on opposite sides of a ruler. Take the ruler away and connect the end points with two segments. Did you form a parallelogram? What about the rest of your group? Fill in the blanks.

**Opposite Sides Congruent and Parallel Theorem:** If one pair of opposite sides of a quadrilateral is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then the quadrilateral is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**4. Proof of Parallelogram Opposite Sides Converse:**

**Given:** In quadrilateral *ABCD***,**and 

**Prove:** *ABCD* is a parallelogram.

Start by drawing .
In ∆*ABC* and ∆*ADC*,  is congruent to \_\_\_\_\_\_\_\_ because it is a shared side.

We are also given that and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

We apply the \_\_\_\_\_\_\_ triangle congruence theorem to conclude that $∆$\_\_\_\_\_\_\_\_\_\_\_\_. Since corresponding parts of congruent triangles are congruent, we can say that
\_\_\_\_\_\_\_\_\_\_ which makes \_\_\_\_\_ ||  by the alternate interior angle converse.

We can also conclude that \_\_\_\_\_\_ by CPCTC.

Thus, \_\_\_\_\_\_\_ || \_\_\_\_\_\_\_ by the alternate interior angle converse.

Therefore, *ABCD* is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ by the definition of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**5.** **Proof of Parallelogram Diagonals Converse:**

**Given:** In quadrilateral *ABCD*,
diagonals and  bisect each other at M.

**Prove:** *ABCD* is a parallelogram.

Since, and bisect each other at M, then it follows that \_\_\_\_\_\_\_ and \_\_\_\_\_\_ by the definition of bisect.

 and are \_\_\_\_\_\_\_\_\_\_\_\_\_ angles, so they must be congruent.

So, $∆$\_\_\_\_\_\_\_\_\_\_\_\_ by the \_\_\_\_\_\_\_\_\_\_ triangle congruence theorem.

Then,  by CPCTC. Complete the proof to show the other pair of opposite sides is congruent:

Since both pairs of opposite sides are congruent, the quadrilateral is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ by the Parallelogram Opposite Sides Converse.

**6.** **Proof of Opposite Sides Congruent and Parallel Theorem**

**Given:** In quadrilateral *ABCD*,and 

**Prove:** *ABCD* is a parallelogram.

Start by drawing .

Since , \_\_\_\_\_\_\_\_\_ by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ theorem.

In ∆*ABC* and ∆*ADC*,  is congruent to \_\_\_\_\_\_\_\_ since it is a shared side.

Since it is also given that , we can conclude that $∆ABC ≅$ $∆ \\_\\_\\_\\_\\_\\_\\_$

by \_\_\_\_\_\_\_\_\_\_\_\_ triangle congruence.

It then follows that by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Since both pairs of sides are congruent, and we just proved the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Converse, *ABCD* must be a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**7.** **If a quadrilateral has two pairs of opposite angles that are congruent, then the quadrilateral is a parallelogram.**

Use the diagram to the right to help prove that *ABCD* is a parallelogram.

y

x

|  |  |
| --- | --- |
| Statements | Reasons |
| 2*x* + 2*y* = 360 |  |
|  | Divide both sides of equation by 2. |
| $∠$ *A* and $∠$ *B* are supplementary.Also $∠$ *A* and $∠$ *D* are supplementary. |  |
| $$\overbar{AD}||\overbar{BC} and \\_\\_\\_\\_\\_\\_||\\_\\_\\_\\_\\_\\_$$ | Same Side Interior Angles Supplementary 🡪Parallel Lines |
|  | Definition of Parallelogram |

**8.** Circle the quadrilaterals that have sufficient evidence to be a parallelogram, based on the markings on the figures.



A) B)



C) D)



 E)