**Activity 3.3.1 Parallel and Perpendicular Lines**



In Algebra 1 you learned that if two lines in the coordinate plane have the same slope, then they are parallel. You also learned that if the slopes of two lines are opposite reciprocals, then the lines are perpendicular. In this activity you will explore the relationship between parallel and perpendicular lines.

1. Here is why parallel lines have the same slope.

In the figure at the right $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD }$are parallel lines intersecting the *x*-axis at points *A* and *C*. *E* and *F* are points on the *x*-axis one unit to the right of *A* and *C*.

*G* lies on $\overleftrightarrow{AB}$ with $\overleftrightarrow{GE}$ perpendicular to the *x*-axis.

*H* lies on $CD$ with $\overleftrightarrow{HF}$ perpendicular to the *x-*axis.

a. $∠$ *GAE* and $∠$ *HCF* are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_angles formed by parallel lines, therefore they are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

b. *AE* = \_\_\_\_\_\_= 1

 c. $∠$*AEG* and$∠$*CFH* are congruent because \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 d. Therefore ∆*AEG* $≅$ ∆*CFH* by the \_\_\_\_\_\_\_ Congruence Theorem.

 e. *EG* = *FH* because \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*EG* = *m*1 and FH = *m*2 Therefore *m*1 = *m*2.

 f. Explain why *m*1 and *m*2 are the slopes of the two parallel lines.

1. Here is why if the slopes of two lines are opposite reciprocals, the lines are perpendicular. (Recall that the product of opposite reciprocals is –1.)

We will demonstrate with a specific example.

In the figure at the right the coordinates of points *A*, *B*, *C*, *D*, and *E* are given.

a. Let *m*1 be the slope of $\overleftrightarrow{AB}$*. m*1 = $\frac{rise}{run}$ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

b. Let *m*2 be the slope of $\overleftrightarrow{AC.} $*m*2 = $\frac{rise}{run}$ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (watch out for signs!)

c. Show that *m*1 *m*2 = –1. (So the slopes are opposite reciprocals.)

Now we need to show that $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are perpendicular.

d. First show that ∆*ADB* $≅$ ∆*CEA*

e. Because corresponding parts of congruent triangles are congruent we know that
m$∠$1 = \_\_\_\_\_\_\_\_ and m$∠$2 = \_\_\_\_\_\_\_\_\_\_

f. By the triangle sum theorem we know that m$∠$1 + m$∠$2 +m$∠$*ADB* = \_\_\_\_\_

g. By the linear pair postulate we know that m$∠$1 + m$∠$*BAC* + m$∠$4 = \_\_\_\_

h. Put (e), (f) and (g) together to show that m$∠$*ADB* = m$∠$*BAC.*

i. Explain why the result in (h) means that $\overleftrightarrow{AB}$ $⊥$ $\overleftrightarrow{AC.}$

3. **Horizontal and Vertical Lines**

Sometimes the slope of a line is not defined.

In the figure at the right,

a. Which line is horizontal?\_\_\_\_\_\_

b. Which line is vertical?\_\_\_\_\_\_\_

c. Which line has zero slope?\_\_\_\_\_\_\_

d. For which line is the slope undefined?\_\_\_\_\_\_

e. Is $\overleftrightarrow{AB}$ $⊥\overleftrightarrow{AC}$?\_\_\_\_\_\_\_ Explain your reasoning.

4. **Perpendicular and Parallel lines.**

In the figure at the right $\overleftrightarrow{CA}⊥$ $\overleftrightarrow{AB}$ and $\overleftrightarrow{DB}$ $⊥\overleftrightarrow{AB}.$

The slope of $\overleftrightarrow{AB}$ = $-\frac{3}{7}$.

a. Find the slope of $\overleftrightarrow{CA}$

b. Find the slope of $\overleftrightarrow{DB}$

c. Explain why we know that $\overleftrightarrow{CA}$ must be parallel to $\overleftrightarrow{DB}.$


5. In the figure at the right,

a. Which lines, if any, are perpendicular?

b. Which lines, if any, are parallel?

c. Explain your reasoning.