**Activity 1.4.2 Inside Changes, Outside Changes**

Often in mathematics, we want to alter a function in some way to match a different but similar situation we want to describe. These alterations are known as *transformations*. It is useful in mathematics to think of two different types of transformations. When we alter the *input* into a function, so that we are changing the independent variable x, we will refer to this as an **inside change** to the function, since when we write f(x), the change occurs “inside the parentheses.” On the other hand, an **outside change** to a function as one where we alter the output from a function, so that we are changing the dependent variable y (the change occurs “outside the parentheses”).

An example might help to clarify. Suppose that you have a function **L(A)** that tells you the number of hours **L(A)** it takes to mow a lawn as a function of **A**, the area of the lawn in square feet.

* If we make an inside change to this function, we are changing the input **A**. For example, **L(A + 10)** is a change of 10 to **A**, the input of the function, the area of the lawn. Therefore, **L(A + 10)** represents the amount of time to mow a lawn with 10 more square feet.
* If we make an outside change to this function, we are changing the output **L(A)**. For example, **L(A) + 10** is a change of 10 to **L(A)**, the output of the function, the number of hours it takes to mow the lawn. Therefore, **L(A) + 10** represents 10 hours more than the amount of time it takes to mow a lawn with area **A**.

1. The cost of attending college **C(r)** is a function of the number of credits taken **r**. Match the following expressions to the verbal descriptions below.  
**C(r + 2)** **C(r) + 2** **C(2r)** **2C(r)**  
  
a. The cost of attending college if the number of credits is doubled:   
  
b. The cost of attending college if there is an additional $2 fee for parking:   
  
c. The cost of attending college if you sign up for two more credits:   
  
d. The cost of attending college if the state doubles the cost:

2. The amount of property taxes you owe your town **T(c)** is a function of the “assessed book value” of your car **c**. Give a mathematical expression that would represent the following:  
  
a. The amount of property tax if the town increases the tax rate by 50%:   
  
b. The amount of property tax if the value of your car increases by 50%:   
  
c. The amount of property tax if the value of your car goes up by $500:   
  
d. The amount of property tax if the town adds an additional fee of $500:

3. The editors of *People* magazine have developed a function that gives a celebrity’s popularity rating, **P(t)**, as a function of the number of “tweets” **t** that are sent about that celebrity on the website [www.twitter.com](http://www.twitter.com). Give a mathematical expression that would represent the following:  
  
a. The popularity rating if the editors add 1000 to everyone’s rating:   
  
b. The popularity rating if a celebrity has 1000 additional tweets:   
  
c. The popularity rating if a celebrity triples the number of tweets:   
  
d. The popularity rating if the editors triple everyone’s rating:

4. The area of a circle, **A(r)**, is a function of its radius **r**. For each of the following, determine if the change is an inside change (to the input) or an outside change (to the output), and then give a verbal description of the change. The first one is done as an example for you.  
  
a. **A(r + 3)**  
 Inside or outside change? *inside*   
  
 Verbal description: *the area of a circle if the radius increases by 3*   
  
b. **A(r) + 3**  
 Inside or outside change?   
  
 Verbal description:   
  
c. **A(2r)**  
 Inside or outside change?   
  
 Verbal description:   
  
d. **2A(r)**  
 Inside or outside change?   
  
 Verbal description:

5. The most common ways for oil companies to keep track of the amount of heating oil its customers need to heat their houses in a year is by keeping track of *degree days*. You can calculate the number of degree days in a day by first calculating the average daily temperature, which is the average of the high and low temperatures that day. If the average is 65 or higher, the number of degree days for that day is 0; and if the average is less than 65, the number of degree days is 65 minus the average daily temperature.  
  
The amount of money paid for heating oil in a year, **H(d)**, is a function of the number of degree days **d** during that year. For each of the following, determine if the change is an inside change (to the input) or an outside change (to the output), and then give a verbal description of the change.  
  
a. **H(1.2d)**  
 Inside or outside change?   
  
 Verbal description:   
  
b. **H(d) + 15**  
 Inside or outside change?   
  
 Verbal description:   
  
c. **1.2H(d)**  
 Inside or outside change?   
  
 Verbal description:   
  
d. **H(d + 15)**  
 Inside or outside change?   
  
 Verbal description:

6. You can think of the population of the world **P(t)** as a function of **t**, the number of years since the year 2000. In the space below, give a verbal description of at least two inside and at least two outside changes to this function. For each one, identify them as either inside or outside changes, and give an expression that matches your verbal description.