**Activity 1.3.3 Piecewise and Step Functions**

A **piecewise function** is a function that is defined by multiple subfunctions on different parts of its domain. Remember that a function is defined as a relation where for each possible value of the input (the domain), there is exactly one output. Since piecewise functions satisfy this definition, it is important to remember that a piecewise function is ONE function, not a collection of functions.

A **step function** is a piecewise function where each piece is a constant function. The graph of a step function will usually resemble a staircase.

The Ironman Triathlon is comprised of three events: a 2.4 mile swim, a 112 mile bicycle ride, and a 26.2 mile run (26.2 miles is the same distance as a regular marathon). The Ironman Triathlon takes about 12 hours for the average triathlete. On average, the 2.4 mile swim is completed in 1.0 hours, the 112 mile bike ride is completed in 6.5 hours, and the 26.2 mile run is completed in 4.5 hours. (Source: <http://www.runtri.com/2011/06/how-long-does-it-take-to-finish-ironman.html>.)

1. Assuming that the speed over each event is constant, find the speed for each event.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Swim = \_\_\_\_\_\_\_\_ | Bike = \_\_\_\_\_\_\_\_ | Run = \_\_\_\_\_\_\_\_ |

2. Make a table of the total distance traveled as a function of time and then graph. Use the color coding to know which speed to use to calculate the distance.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  | | --- | --- | | **Time in hours**  **t** | **Distance in miles**  **D(t) = speed × time** | | 0.5 |  | | 1.0 |  | | 1.5 |  | | 2.0 |  | | 2.5 |  | | 3.0 |  | | 3.5 |  | | 4.0 |  | | 4.5 |  | | 5.0 |  | | 5.5 |  | | 6.0 |  | | 6.5 |  | | 7.0 |  | | 7.5 |  | | 8.0 |  | | 8.5 |  | | 9.0 |  | | 9.5 |  | | 10.0 |  | | 10.5 |  | | 11.0 |  | | 11.5 |  | | 12.0 |  | |  |

3. Find the slope of each line using the graph.

|  |  |  |
| --- | --- | --- |
| Swim = \_\_\_\_\_\_\_\_ | Bike = \_\_\_\_\_\_\_\_ | Run = \_\_\_\_\_\_\_\_ |

4. What do you notice about the slope and the speed?

5. What was the general shape of the graph?

6. What kind of function would you say the graph/table represent?

7. Below is an income tax table. Create a graph from this table, with income as the independent variable and tax as the dependent variable.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  | | --- | --- | --- | | **At Least** | **Less Than or Equal To** | **Tax** | | $0 | $19,999 | $0 | | $20,000 | $29,999 | $113 | | $30,000 | $39,999 | $683 | | $40,000 | $49,999 | $1598 | | $50,000 | $59,999 | $2095 | | $60,000 | $69,999 | $2878 | | $70,000 | $79,999 | $3488 | |  |

8. Describe the shape of the function graphed.

9. What kind of function is graphed from the table above?

10. Below is a graph that represents how much shipping cost from UPS based on weight.

11. What is the slope between 1 and 2 pounds and the slope between 2 and 3?

12. What does the slope represent?

13. What was the general shape of the graph?

14. What kind of function would you say the graph represents?

In mathematics, the greatest integer function of x (also sometimes called the floor function of x) is defined as the largest integer less than or equal to x. The symbol for greatest integer of x is f(x) = ⌊x⌋. For a positive number, ⌊x⌋ is the “integer part” of x; for a negative number, ⌊x⌋ is the next integer smaller than x. For example, ⌊1.63⌋ = 1, ⌊1⌋ = 1, and ⌊-1.63⌋ = -2.

15. Find the following values:

a. ⌊6.75⌋ d. ⌊-3⌋

b. ⌊¾⌋ e. ⌊-π⌋

c. ⌊274⌋ f. ⌊6.23 – 2.8914⌋

16. Complete the following table:

|  |  |  |
| --- | --- | --- |
| If x is greater than or equal to | And x is less than | Then ⌊x⌋ equals |
| 3 | -2 | -3 |
| -2 | -1 |  |
| -1 | 0 |  |
| 0 | 1 |  |
| 1 | 2 |  |
| 2 | 3 |  |

17. Based on your table in #16, sketch a graph of f(x) = ⌊x⌋ on the coordinate axes below.



18. What kind of function is the greatest integer function?