**Activity 1.3.2 The Absolute Value Function**

**Definition:** The **absolute value** of a number x is written as |x| and is defined as follows:

$|x|=\left\{\begin{array}{c}-x if x<0\\x if x\geq 0\end{array}\right.$.

In other words, x is the non-negative value of x, without regard to its sign. It is useful to think of |x| as being the distance between x and the number 0.

1. Graph the absolute value functions by using a table.



a. 

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | *f* (*x*) |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

 |  |

b. 

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | *f* (*x*) |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

 |  |

c. 

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | *f* (*x*) |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

 |  |

d. 



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | *f* (*x*) |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

 |  |

e. 

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| *x* | *f* (*x*) |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

 |  |

2. Graph the absolute value functions by using a graphing calculator.

|  |  |
| --- | --- |
| a. b.  | **Steps on a TI- 84**1. Click Y = screen2. Press MATH, NUM, 1:abs(3. Enter the function in after abs(4. Click GraphFor example for , enter Y1 = abs(x-3) |

Suppose that you are the town manager of Pleasant Ville. Your town needs to build a fire station that will be in the best location possible. What location is “best”? One way to answer this question is to locate the fire station so that it is as close to all of the buildings in town as possible. But if you think about any town with more than one building, if you move closer to some buildings you are moving farther away from others. So the question remains, what location is “best”?

Let’s make a simple mathematical model of this situation. Let’s assume all the houses are placed on a straight line, at points A, B, C, etc. Let’s also assume that Pleasant Ville only uses one fire truck, that there is only one fire at a time, and that all the buildings are equally likely to have a fire. Under these conditions, we can use absolute values to answer this question.

3. Let’s start with only two houses in Pleasant Ville. Ashley lives at the point A = -1 on the number line, and Bobby lives at the point B = 3. Then the distance to the origin from A is a(x) = |x + 1|, and the distance from the origin to B is b(x) = |x – 3|.

a. Complete the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| x | a(x) = |x + 1| | b(x) = |x – 3| | a(x) + b(x) |
| -8 |  |  |  |
| -7 |  |  |  |
| -6 |  |  |  |
| -5 |  |  |  |
| -4 |  |  |  |
| -3 |  |  |  |
| -2 |  |  |  |
| -1 |  |  |  |
| 0 |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |

b. What is the smallest total distance possible (the smallest value of a(x) + b(x))? Where does that value occur?

c. On the set of axes below, sketch a graph of a(x) + b(x).



4. a. Ashley and Bobby move so that Ashley now lives at the point A = -6 and Bobby lives at the point B = 5. Complete the table below for Ashley and Bobby’s new locations.

|  |  |  |  |
| --- | --- | --- | --- |
| x | a(x) = |x + 6| | b(x) = |x – 5| | a(x) + b(x) |
| -8 |  |  |  |
| -7 |  |  |  |
| -6 |  |  |  |
| -5 |  |  |  |
| -4 |  |  |  |
| -3 |  |  |  |
| -2 |  |  |  |
| -1 |  |  |  |
| 0 |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |

b. What is the smallest total distance possible (the smallest value of a(x) + b(x))? Where does that value occur?

c. On the set of axes below, sketch a graph of a(x) + b(x).



5. It turns out that the pattern you see in questions #3 and #4 is true any time you have two buildings in town. See if you can describe that pattern by completing the following sentence:

If there are two buildings in town built at the points x = A and x = B, then the fire station

should be built .

6. a. Our model becomes a little more complicated (and a little more realistic) when Charlise moves into town. Ashley and Bobby are now back in their original houses, Ashley at A = -1 and Bobby at B = 3, when Charlise moves in at C = 6. Complete the table below for Ashley, Bobby, and Charlise.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | a(x) = |x + 1| | b(x) = |x – 3| | c(x) = |x – 6| | a(x) + b(x) + c(x) |
| -8 |  |  |  |  |
| -7 |  |  |  |  |
| -6 |  |  |  |  |
| -5 |  |  |  |  |
| -4 |  |  |  |  |
| -3 |  |  |  |  |
| -2 |  |  |  |  |
| -1 |  |  |  |  |
| 0 |  |  |  |  |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
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| 7 |  |  |  |  |
| 8 |  |  |  |  |

b. What is the smallest total distance possible (the smallest value of a(x) + b(x)) + c(x))? Where does that value occur?

c. On the set of axes below, sketch a graph of a(x) + b(x) + c(x).



7. Ashley moves back to A = -6 and Bobby to B = 5, and Charlise moves between them at

C = 2. Complete the table below for Ashley, Bobby, and Charlise.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | a(x) = |x + 6| | b(x) = |x – 5| | c(x) = |x – 2| | a(x) + b(x) + c(x) |
| -8 |  |  |  |  |
| -7 |  |  |  |  |
| -6 |  |  |  |  |
| -5 |  |  |  |  |
| -4 |  |  |  |  |
| -3 |  |  |  |  |
| -2 |  |  |  |  |
| -1 |  |  |  |  |
| 0 |  |  |  |  |
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| 7 |  |  |  |  |
| 8 |  |  |  |  |

b. What is the smallest total distance possible (the smallest value of a(x) + b(x)) + c(x))? Where does that value occur?

c. On the set of axes below, sketch a graph of a(x) + b(x) + c(x).



8. Once again, it turns out that the pattern in questions #6 and #7 is true any time you have three buildings in town. Describe that pattern by completing the following sentence:

If there are three buildings in town built at the points x = A, x = B, and x = C, then the fire

station should be built .

9. a. Diego has decided he likes Pleasant Ville and moves in. Ashley, Bobby, and Charlise are happy in their homes at A = -6, B = 5, and C = 2, and Diego moves in near Ashley at D = -7. Complete the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | a(x) = |x + 6| | b(x) = |x – 5| | c(x) = |x – 2| | d(x) = |x + 7| | a(x) + b(x) + c(x) + d(x) |
| -8 |  |  |  |  |  |
| -7 |  |  |  |  |  |
| -6 |  |  |  |  |  |
| -5 |  |  |  |  |  |
| -4 |  |  |  |  |  |
| -3 |  |  |  |  |  |
| -2 |  |  |  |  |  |
| -1 |  |  |  |  |  |
| 0 |  |  |  |  |  |
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| 8 |  |  |  |  |  |

b. What is the smallest total distance possible (the smallest value of a(x) + b(x)) + c(x) + d(x))? Where does that value occur?

c. On the set of axes below, sketch a graph of a(x) + b(x) + c(x) +d(x).



10. Based on questions #3 through #9, you might have a *conjecture* for what happens in this problem in general. What is your conjecture about what happens when we have an even number of houses? What is your conjecture about what happens when we have an odd number of houses?